Doctoral Thesis Defense

Signal Processing Methods for Large-Scale Multi-Antenna Systems

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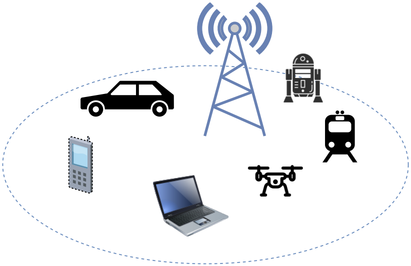
Advisor: Prof. Dr. André Lima Férrer de Almeida
Co-Advisor: Prof. Dr. João César Moura Mota

Universidade Federal do Ceará
Teleinformatics Engineering Department

Fortaleza, October 10th, 2019

Motivation and Scope



- Number of connected devices keeps growing every year
- Very large capacity requirements
- How to achieve larger system capacity?
  - **Beamforming gain** → Massive MIMO
  - **Increase bandwidth** → Millimeter wave bands

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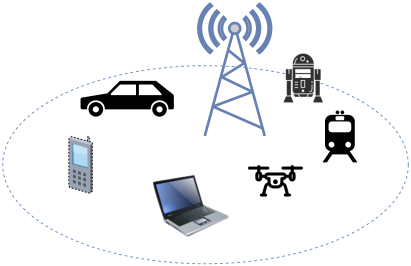

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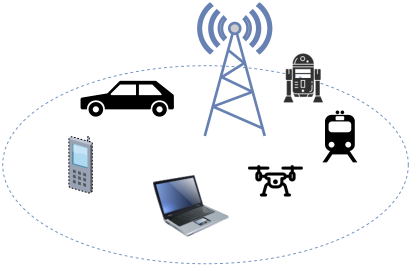

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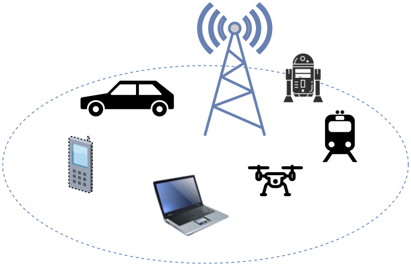

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Motivation and Scope

Challenges

1. Computational complexity of large-scale filter design;
2. Energy efficiency of mmWave massive MIMO transceivers;
3. MmWave channel estimation under synchronization impairments.

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State of the Art – Problem 1

Iterative implementation of linear filters

- P. Harris et al., *"Serving* 22 *users in real-time with a* 128*-antenna massive MIMO testbed."* 2016 IEEE International Workshop on Signal Processing Systems (SiPS), p. 266-272.
  - Systolic array implementation of QR decomposition for Zero-Forcing filtering
- X. Qin et al., *"A near-optimal detection scheme based on joint steepest descent and Jacobi method for uplink massive MIMO systems,"* IEEE Communications Letters, v. 20, n. 2, p. 276-279, 2015.
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State of the Art – Problem 2

Hybrid analog/digital (A/D) systems

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Digital systems with low-resolution data converters

K. Roth et al., *"A comparison of hybrid beamforming and digital beamforming with low-resolution ADCs for multiple users and imperfect CSI,"* IEEE Journal of Selected Topics in Signal Processing, v. 12, n. 3, p. 484-498, 2018.

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State of the Art – Problem 3

MmWave channel estimation with carrier frequency offset (CFO) impairment

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  - Sparse bilinear optimization $\rightarrow$ message passing solution
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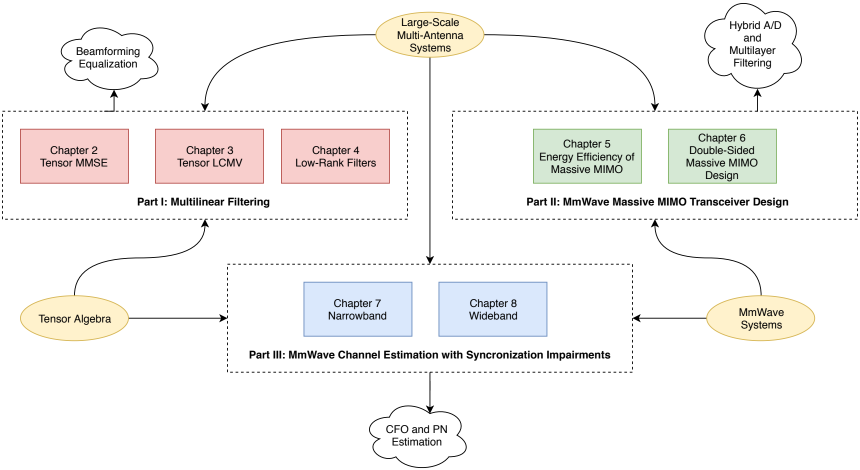
Thesis Overview



Part I: Multilinear Filtering

Related publications

- *IET Signal Processing, v. 13, n. 4, p. 434–442, June 2019*
- *Signal Processing, v. 158, p. 15–25, May 2019*
- *Proc. SBRT 2018*
- *Proc. IEEE ISWCS 2019*

Multilinear Filtering

- Multi-linear and time-invariant filter:

$$\boldsymbol{w} = \boldsymbol{w}_1 \otimes \cdots \otimes \boldsymbol{w}_M \in \mathbb{C}^N$$

  where $\boldsymbol{w}_m \in \mathbb{C}^{N_m}$ with $\prod_{m=1}^{M} N_m = N$

- Basic idea: design **each** factor instead of the **whole** vector
- Fewer computations?
- How much performance loss, if any?
- **Beamforming and equalization problems**

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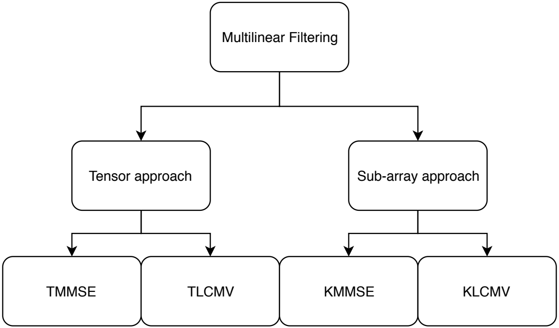

Scenario

- Narrowband far-field propagation
- R independent sources $s_r[k]$ impinging on the receiver with N antennas
- Multi-user system with R users and line-of-sight propagation

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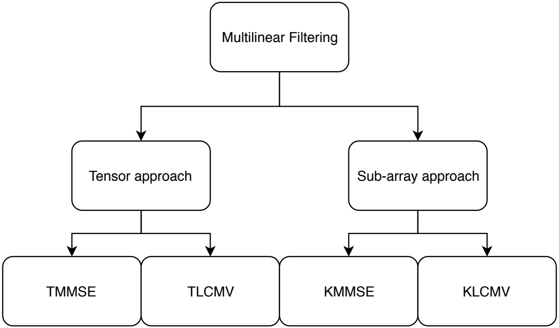

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System Model

Received Signal

$$\boldsymbol{x}[k] = \boldsymbol{A}\boldsymbol{s}[k] + \boldsymbol{b}[k] \quad (1)$$

- $\boldsymbol{s}[k] = [s_1[k], \ldots, s_R[k]]^{\mathsf{T}} \in \mathbb{C}^R$ – sources vector
- $\boldsymbol{A} = [\boldsymbol{a}(\phi_1, \theta_1), \ldots, \boldsymbol{a}(\phi_R, \theta_R)] \in \mathbb{C}^{N \times R}$ – array manifold matrix
- $\boldsymbol{b}[k] = [b_1[k], \ldots, b_N[k]]^{\mathsf{T}} \in \mathbb{C}^N$ – ad. white Gaus. noise (AWGN)

Beamforming Filter

- Filter $\boldsymbol{x}[k]$ to recover a signal of interest ($r = 1$)
- $\boldsymbol{w} = [w_1, \ldots, w_N]^{\mathsf{T}} \in \mathbb{C}^N$
- Filter output:

$$y[k] = \boldsymbol{w}^{\mathsf{H}}\boldsymbol{x}[k]$$

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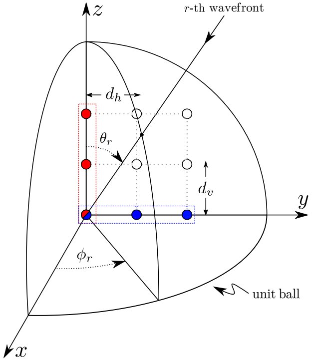
Uniform Planar Array



UPA array response is **separable**

$$\boldsymbol{a}(\phi_r, \theta_r) = \begin{bmatrix} 1 \\ e^{-j\pi\cos\theta_r} \\ \vdots \\ e^{-j\pi(N_v-1)\cos\theta_r} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ e^{-j\pi\sin\phi_r\sin\theta_r} \\ \vdots \\ e^{-j\pi(N_h-1)\sin\phi_r\sin\theta_r} \end{bmatrix}$$
$$= \boldsymbol{a}_v(q_r) \otimes \boldsymbol{a}_h(p_r)$$

where $p_r = \sin\phi_r \sin\theta_r$ and $q_r = \cos\theta_r$.

Array manifold matrix:

$$\boldsymbol{A} = \boldsymbol{A}_v \diamond \boldsymbol{A}_h \in \mathbb{C}^{N_v N_h \times R}$$

Apply separable filter $w = w_v \otimes w_h$ to each array dimension

$$(\boldsymbol{w}_v \otimes \boldsymbol{w}_h)^{\mathsf{H}}(\boldsymbol{A}_v \diamond \boldsymbol{A}_h) = (\boldsymbol{w}_v^{\mathsf{H}}\boldsymbol{A}_v) \otimes (\boldsymbol{w}_h^{\mathsf{H}}\boldsymbol{A}_h)$$

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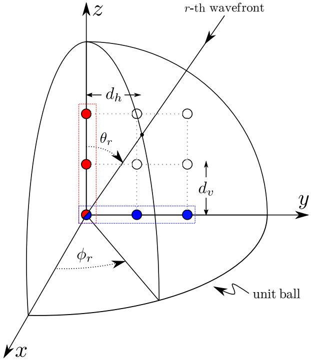

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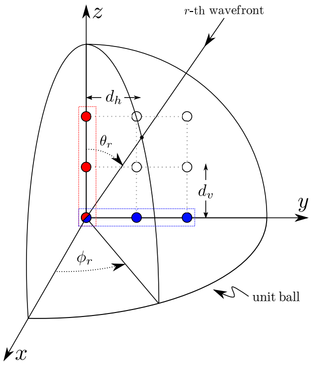

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System Model – Tensor Formulation

- We define the array steering tensor

$$\mathcal{A} = \mathcal{I}_{3,R} \times_1 \boldsymbol{A}_h \times_2 \boldsymbol{A}_v \times_3 \boldsymbol{I}_R \in \mathbb{C}^{N_h \times N_v \times R} \quad (2)$$

- Received signal model

$$\boldsymbol{X}[k] = \mathcal{A} \times_3 \boldsymbol{s}^{\mathsf{T}}[k] + \boldsymbol{B}[k] \in \mathbb{C}^{N_h \times N_v} \quad (3)$$

- Filter $\boldsymbol{w} = \boldsymbol{w} = \boldsymbol{w}_v \otimes \boldsymbol{w}_h$ output:

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- Define $\boldsymbol{u}_h[k] = \boldsymbol{X}[k]\boldsymbol{w}_v^* \in \mathbb{C}^{N_h}$ and $\boldsymbol{u}_v[k] = \boldsymbol{X}^\mathsf{T}[k]\boldsymbol{w}_h^* \in \mathbb{C}^{N_v}$
- Output signal rewritten as

$$y[k] = \boldsymbol{w}_h^\mathsf{H} \boldsymbol{u}_h[k] = \boldsymbol{w}_v^\mathsf{H} \boldsymbol{u}_v[k] \tag{6}$$

- **Output bilinear w.r.t. sub-filters**

Beamforming Filter Design – Tensor MMSE (TMMSE)

- Consider the classical minimum mean square error (MMSE) filter design:

$$\min_{\boldsymbol{w}} \mathbb{E}\left[\left|s_{\mathrm{SOI}}[k] - \boldsymbol{w}^{\mathsf{H}}\boldsymbol{x}[k]\right|^2\right] \tag{7}$$

- From the bilinearity property, we may write

$$\min_{\boldsymbol{w}_h} \mathbb{E}\left[\left|s_{\mathrm{SOI}}[k] - \boldsymbol{w}_h^{\mathsf{H}}\boldsymbol{u}_h[k]\right|^2\right] \tag{8a}$$

$$\min_{\boldsymbol{w}_v} \mathbb{E}\left[\left|s_{\mathrm{SOI}}[k] - \boldsymbol{w}_v^{\mathsf{H}}\boldsymbol{u}_v[k]\right|^2\right] \tag{8b}$$

- Alternating optimization in (8a) and (8b) until convergence
- After convergence[1]: $\boldsymbol{w}_{\mathrm{TMMSE}} = \boldsymbol{w}_v \otimes \boldsymbol{w}_h$
- Tikhonov regularization is applied to avoid numerical instability
- Exchange **degrees of freedom** for complexity reduction
- N (linear) vs. $\min(N_h, N_v)$ (tensor)

[1] A. Yener, R. D. Yates, S. Ulukus, *"Interference management for CDMA systems through power control, multiuser detection, and beamforming,"* IEEE Transactions on Communications, v. 49, n. 7, p. 1227–1239, 2001.

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Beamforming Filter Design – Tensor LCMV (TLCMV)

- We also consider the linear constraint minimum variance (LCMV) filter

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathsf{H}} \boldsymbol{R}_{xx} \boldsymbol{w}, \quad \text{s.t. } \boldsymbol{C}^{\mathsf{H}} \boldsymbol{w} = \boldsymbol{f} \tag{9}$$

  where $\boldsymbol{C} \in \mathbb{C}^{N \times R}$ denotes the constraint matrix, $\boldsymbol{f} \in \mathbb{C}^{R}$ the array factor vector and $\boldsymbol{R}_{xx}$ the cov. matrix of $\boldsymbol{x}[k]$

- We can decouple (9) into

$$\min_{\boldsymbol{w}_h} \boldsymbol{w}_h^{\mathsf{H}} \boldsymbol{R}_{hh} \boldsymbol{w}_h, \quad \text{s.t. } \boldsymbol{C}_h^{\mathsf{H}} \boldsymbol{w}_h = \boldsymbol{f}_h \tag{10a}$$

$$\min_{\boldsymbol{w}_v} \boldsymbol{w}_v^{\mathsf{H}} \boldsymbol{R}_{vv} \boldsymbol{w}_v, \quad \text{s.t. } \boldsymbol{C}_v^{\mathsf{H}} \boldsymbol{w}_v = \boldsymbol{f}_v \tag{10b}$$

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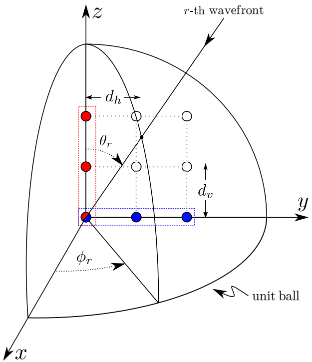
System Model – Sub-array Formulation



- Linear sub-arrays in planar array
- Horizontal sub-array

$$\boldsymbol{x}_h[k] = \boldsymbol{A}_h \boldsymbol{s}[k] + \boldsymbol{b}_h[k] \in \mathbb{C}^{N_h} \quad (11)$$

- Vertical sub-array

$$\boldsymbol{x}_v[k] = \boldsymbol{A}_v \boldsymbol{s}[k] + \boldsymbol{b}_v[k] \in \mathbb{C}^{N_v} \quad (12)$$

- **Idea:** design $\boldsymbol{w}_h$ and $\boldsymbol{w}_v$ independently
- **Capture sub-array signals only**
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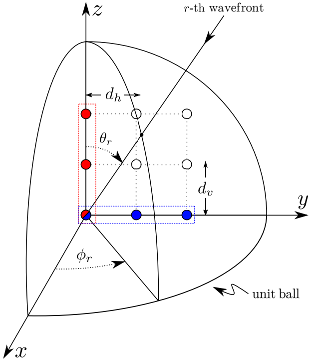

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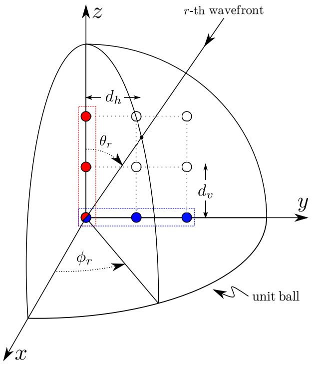

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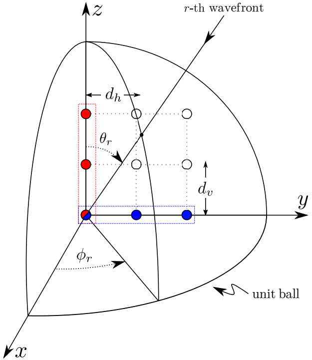

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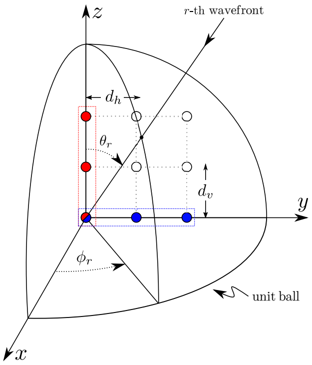

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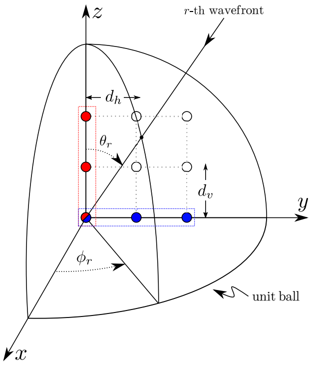

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Beamforming Filter Design – Kronecker Filters

Kronecker MMSE (KMMSE) Filter

$$\min_{\boldsymbol{w}_h} \mathbb{E}\left[\left|s_{\mathrm{SOI}}[k] - \boldsymbol{w}_h^{\mathsf{H}} \boldsymbol{x}_h[k]\right|^2\right] \tag{13a}$$

$$\min_{\boldsymbol{w}_v} \mathbb{E}\left[\left|s_{\mathrm{SOI}}[k] - \boldsymbol{w}_v^{\mathsf{H}} \boldsymbol{x}_v[k]\right|^2\right] \tag{13b}$$

Kronecker LCMV (KLCMV) Filter

$$\min_{\boldsymbol{w}_h} \boldsymbol{w}_h^{\mathsf{H}} \boldsymbol{R}_h \boldsymbol{w}_h, \quad \text{s.t. } \boldsymbol{C}_h^{\mathsf{H}} \boldsymbol{w}_h = \boldsymbol{f}_h \tag{14a}$$

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where $\boldsymbol{R}_h$ and $\boldsymbol{R}_v$ are the covariance matrices of $\boldsymbol{x}_h[k]$ and $\boldsymbol{x}_v[k]$, respectively

Compute w_h, w_v and combine with Kronecker once!

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Beamforming Filter Design

Computational Complexity

- MMSE/LCMV: $O(N^3)$
- TMMSE/TLCMV: $O(I(N_h^3 + N_v^3))$, for I iterations
- KMMSE/KLCMV: $O(N_h^3 + N_v^3)$

- The MMSE and LCMV filters (as well as their tensor extensions) depend on second-order statistics
- **Sample estimates when they are not known**
- The adaptive implementation of the proposed tensor and Kronecker MMSE and LCMV filters have been developed

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Simulation Results

Setup

- Direction cosines p_r and q_r uniformly distributed in $\mathcal{U}(-0.9,\, 0.9)$
- $R = 4$ sources QPSK signals
- $N = 64$ antennas ($N_h = N_v = 8$), half-wave spacing

Figures of Merit

- Floating point operations (flops) – computational complexity
- Uncoded bit error ratio (BER) for MMSE-type filters
- Output SINR for LCMV-type filters

$$\mathrm{SINR}_{\mathrm{out}} = \frac{\boldsymbol{w}^{\mathsf{H}} \boldsymbol{R}_{dd} \boldsymbol{w}}{\boldsymbol{w}^{\mathsf{H}} (\boldsymbol{R}_{ii} + \boldsymbol{R}_{bb}) \boldsymbol{w}}.$$

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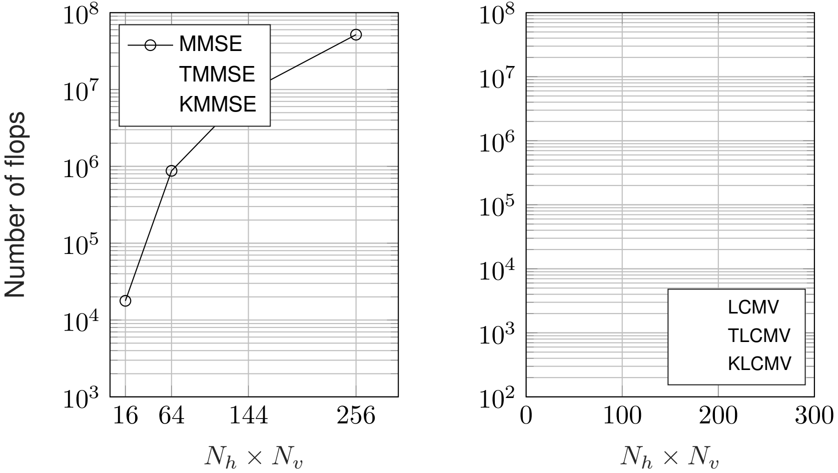

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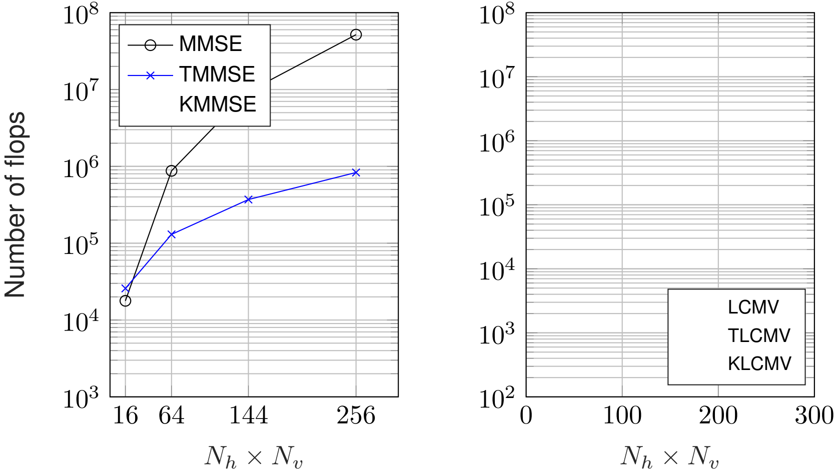
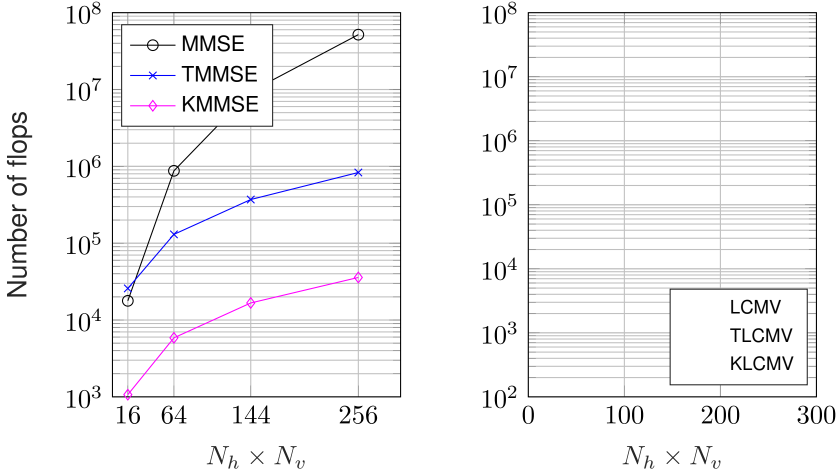


Simulation Results – Computational Complexity



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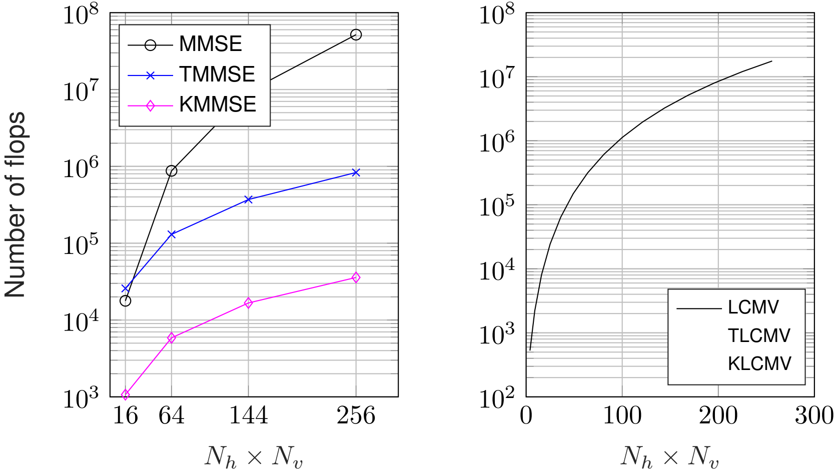

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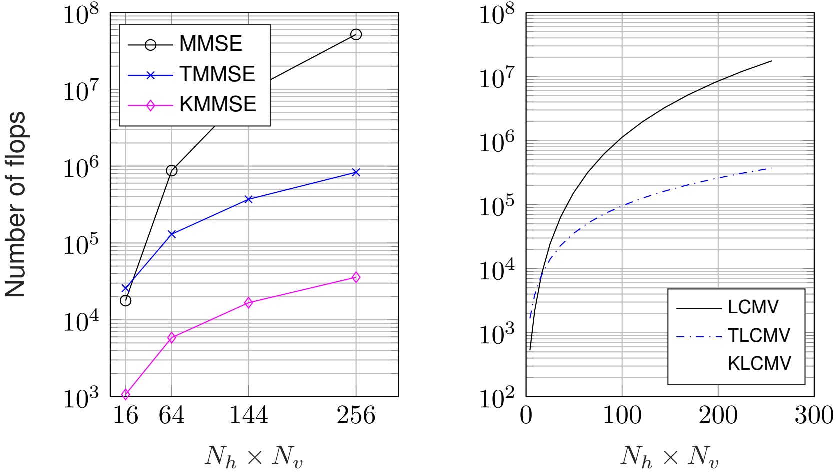

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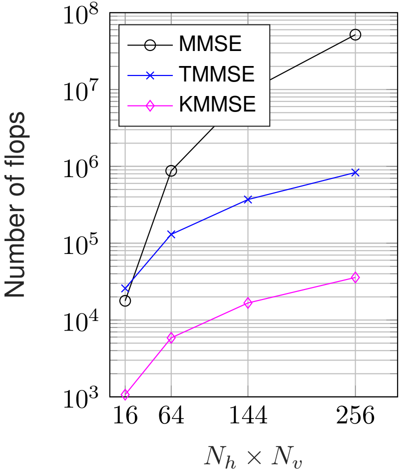
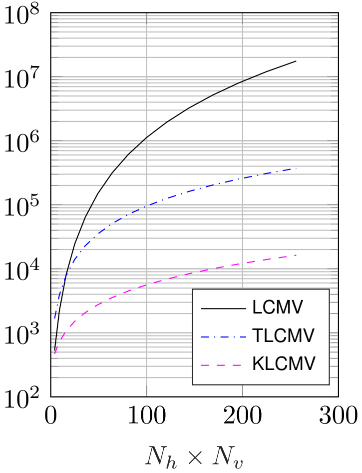
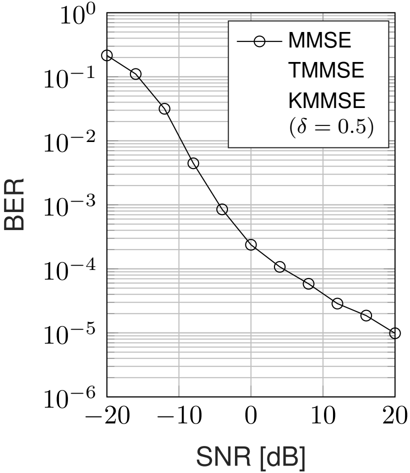
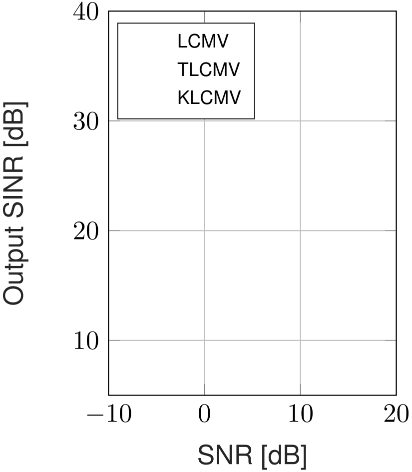




Simulation Results – BER and SINR





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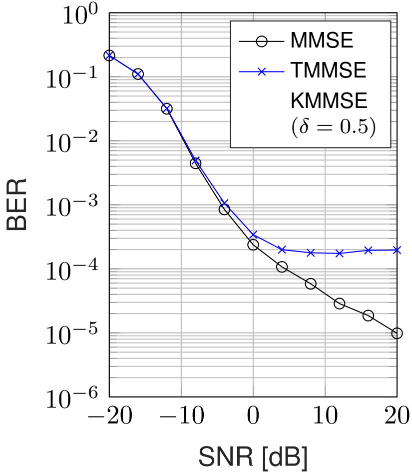

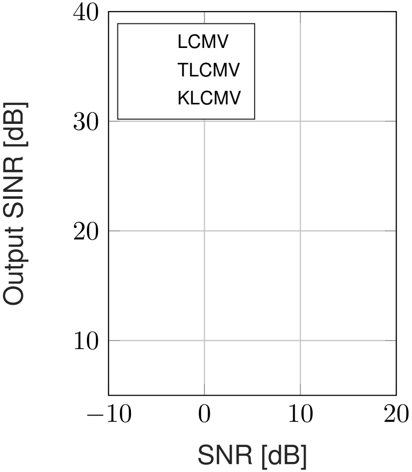

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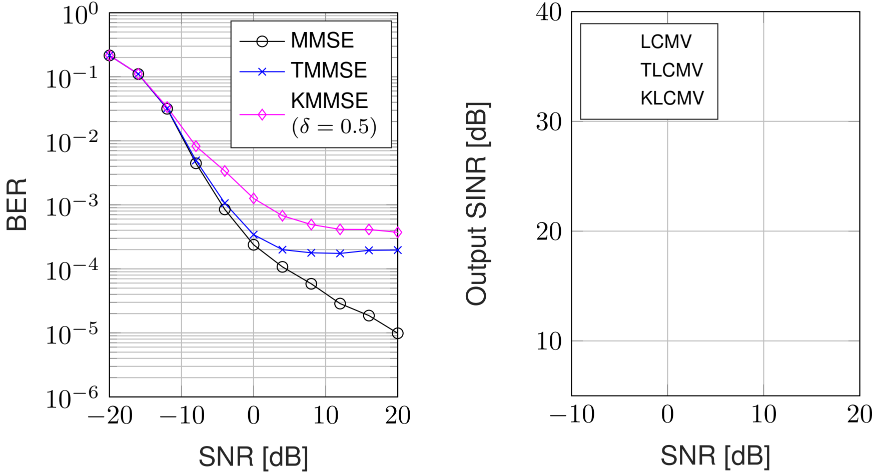

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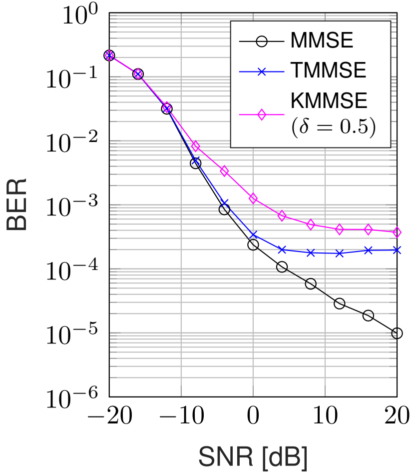

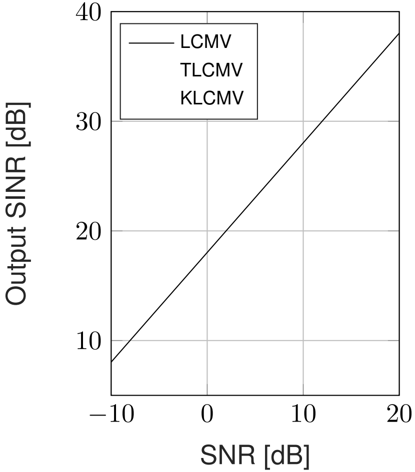

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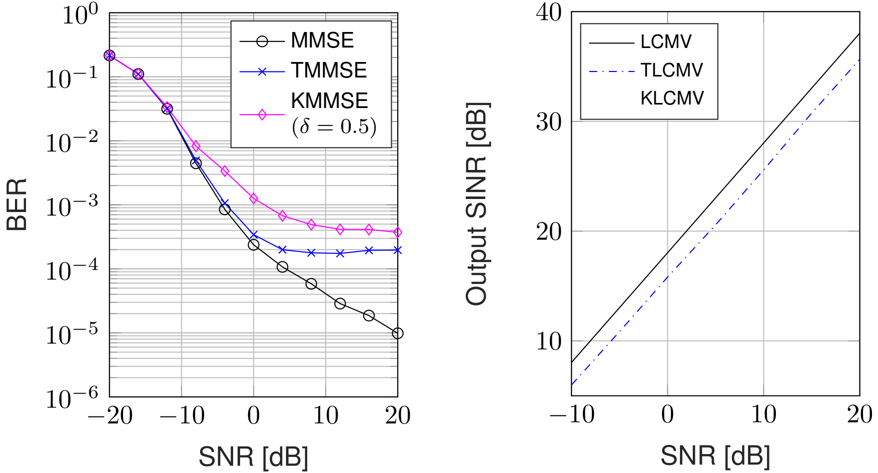

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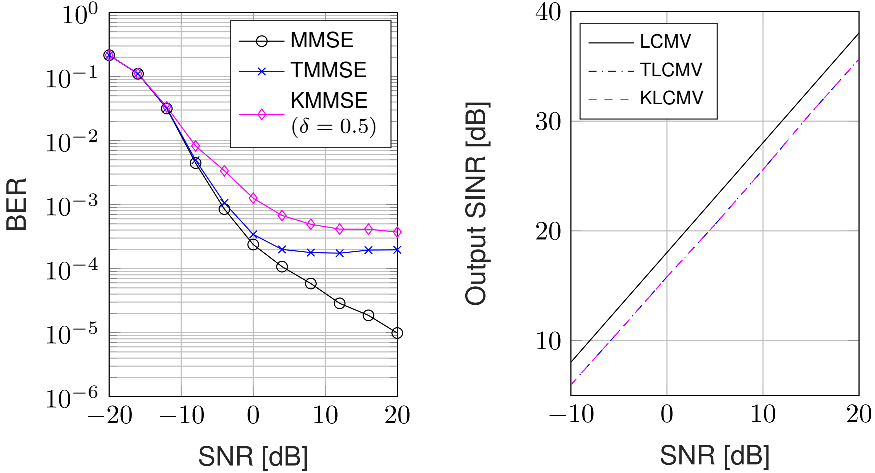

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Multipath?

Low-Rank Filters

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System Model

- Uplink scenario, U users

$$\boldsymbol{x}[k] = \sum_{u=1}^{U} \boldsymbol{H}_u \boldsymbol{s}_u[k] + \boldsymbol{b}[k] \tag{15}$$

$$\boldsymbol{s}_u[k] = [s_u[k], \ldots, s_u[k-Q+1]]^\mathsf{T} \tag{16}$$

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$$\boldsymbol{a}(\theta_{u,\ell}) = \left[1, \ldots, e^{-\jmath\pi(N-1)\cos\theta_{u,\ell}}\right]^\mathsf{T} \in \mathbb{C}^{N} \tag{18}$$

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Some Algebra...

The filter coefficients can be written as

$$w_{n_1,\ldots,n_D} = \sum_{r=1}^{R} \prod_{d=1}^{D} [\boldsymbol{w}_{d,r}]_{n_d}, \tag{20}$$

which allows us to recast the equalizer output $y[k] = \boldsymbol{w}^{\mathsf{H}} \boldsymbol{x}[k]$ as follows

$$y[k] = \sum_{n_1,\ldots,n_D=1}^{N_1,\ldots,N_D} \left(\sum_{r=1}^{R} [\boldsymbol{w}_{1,r}]_{n_1}^{*} \ldots [\boldsymbol{w}_{D,r}]_{n_D}^{*} \right) x_{n_1,\ldots,n_D}[k]. \tag{21a}$$

$$= \sum_{r=1}^{R} \sum_{n_d=1}^{N_d} [\boldsymbol{w}_{d,r}]_{n_d}^{*} \left(\sum_{n_q=1}^{N_q} \prod_{q \neq d}^{D} [\boldsymbol{w}_{q,r}]_{n_q}^{*} x_{n_1,\ldots,n_D}[k] \right) \tag{21b}$$

$$= \sum_{r=1}^{R} \sum_{n_d=1}^{N_d} [\boldsymbol{w}_{d,r}]_{n_d}^{*} [\boldsymbol{u}_{d,r}[k]]_{n_d} = \boldsymbol{w}_d^{\mathsf{H}} \boldsymbol{u}_d[k] \tag{21c}$$

Output is linear w.r.t. each tensor filter factor w_d!

Low-Rank Tensor MMSE

- We formulate for each filter mode

$$\min_{\boldsymbol{w}_d} \mathbb{E}\left[|s_u[k-\delta] - \boldsymbol{w}_d^{\mathsf{H}}\boldsymbol{u}_d[k]|^2\right], \quad d \in \{1,\ldots,D\}.$$

where

$$\boldsymbol{u}_d[k] = \left[\boldsymbol{u}_{d,1}^{\mathsf{T}}[k],\ldots,\boldsymbol{u}_{d,R}^{\mathsf{T}}[k]\right]^{\mathsf{T}} \in \mathbb{C}^{RN_d} \tag{22}$$

$$\boldsymbol{u}_{d,r}[k] = \boldsymbol{X}_{(d)}[k] \bigotimes_{q\neq d}^{D} \boldsymbol{w}_{q,r}^{*} \in \mathbb{C}^{N_d} \tag{23}$$

$$\boldsymbol{w}_d = \left[\boldsymbol{w}_{d,1}^{\mathsf{T}},\ldots,\boldsymbol{w}_{d,R}^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{C}^{RN_d} \tag{24}$$

- Solution:

$$\boldsymbol{w}_{d,\text{MMSE}} = \boldsymbol{R}_{u_d,u_d}^{-1}\boldsymbol{p}_{u_d} \in \mathbb{C}^{RN_d}, \tag{25}$$

$$\boldsymbol{R}_{u_d,u_d} = \mathbb{E}\left[\boldsymbol{u}_d[k]\boldsymbol{u}_d^{\mathsf{H}}[k]\right] \in \mathbb{C}^{RN_d\times RN_d}, \tag{26}$$

$$\boldsymbol{p}_{u_d} = \mathbb{E}\left[\boldsymbol{u}_d[k]s_u^{*}[k-\delta]\right] \in \mathbb{C}^{RN_d} \tag{27}$$

- **Alternating optimization**

Computational Complexity

- N: number of antennas
- K: number of snapshots (covariance matrix estimation)
- MMSE filter

$$P_{\text{MMSE}}(N,K) = \underbrace{N^2K + NK}_{\text{statistics estimation}} + \overbrace{O(N^3)}^{\text{cov. matrix inversion}} + \underbrace{N^2}_{\text{filtering}}$$

- LR-TMMSE filter

$$P_{\text{LR-TMMSE}}(\{N_d\}, D, I, K) =$$
$$I\left[\sum_{d=1}^{D} \underbrace{R(D-1)NK + N_d^2K + N_dK}_{\text{statistics estimation}} + \overbrace{O(N_d^3)}^{\text{cov. matrix inversion}} + \underbrace{N_d^2}_{\text{filtering}}\right]$$

- I: iterations number
- **Tensor overhead!**

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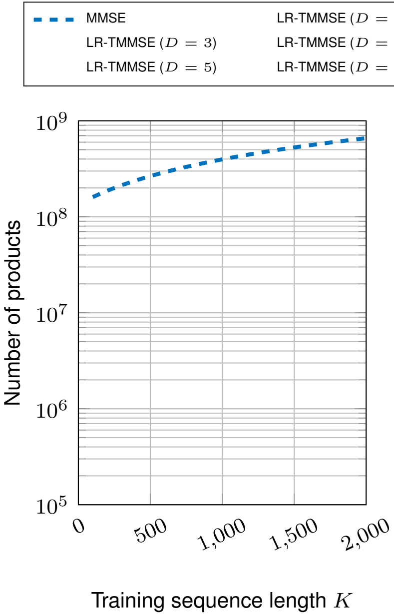
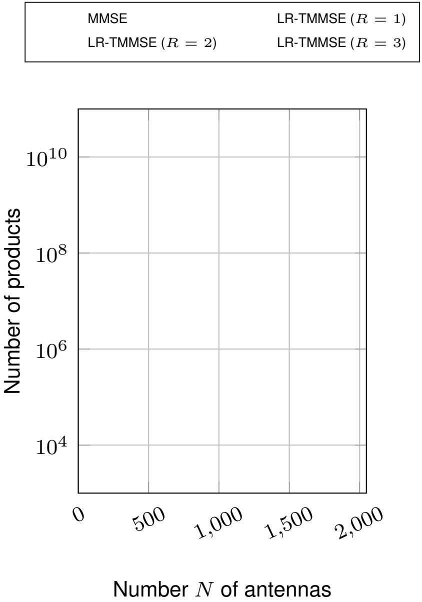
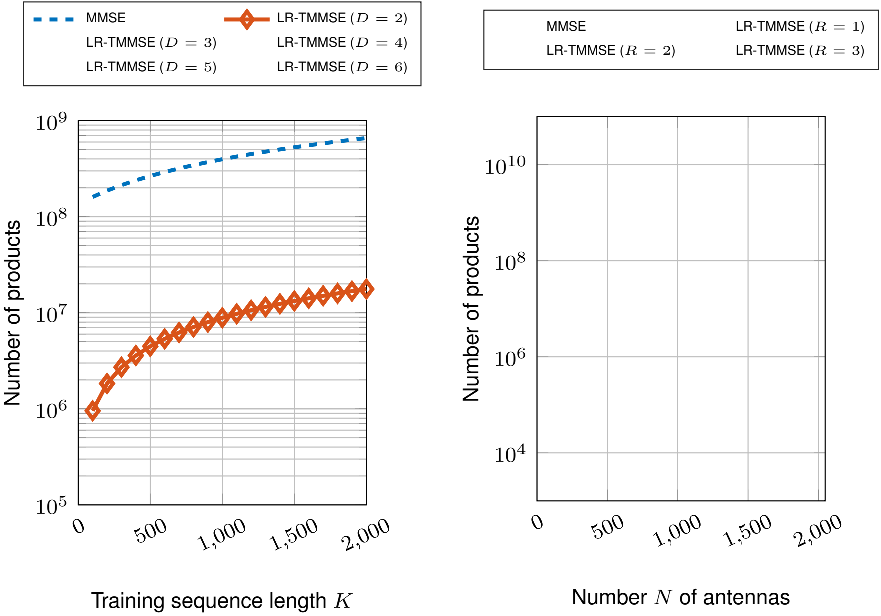


$N = 512$, $I = 2$, $R = 3$



$K = 600$, $I = 2$, $D = 3$



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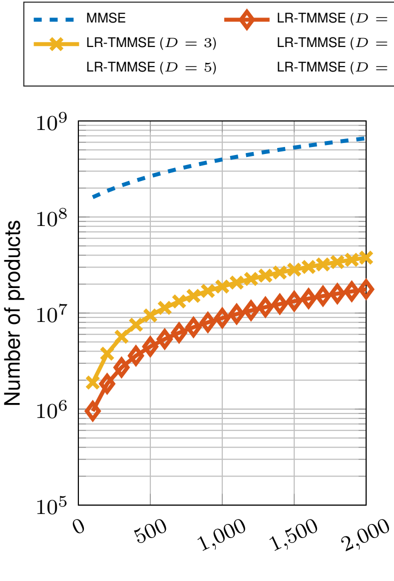

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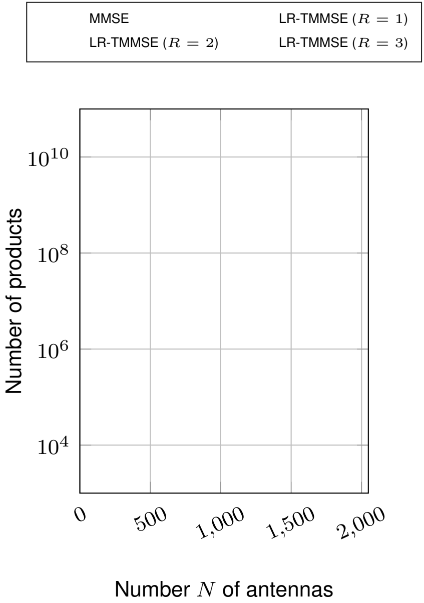

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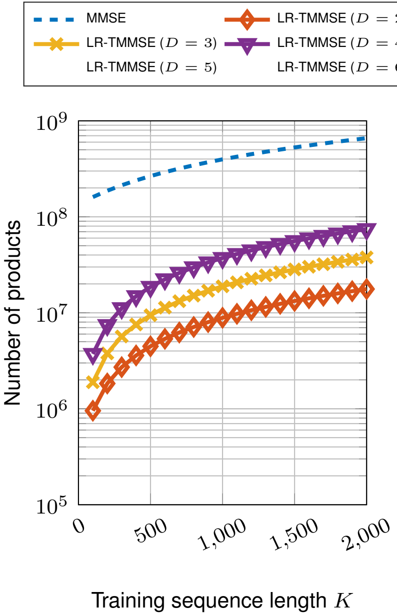

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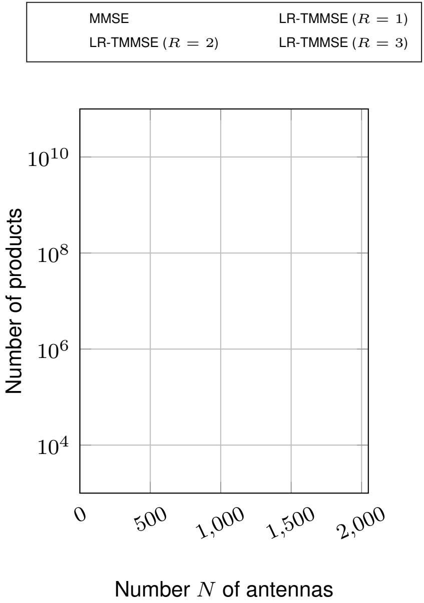

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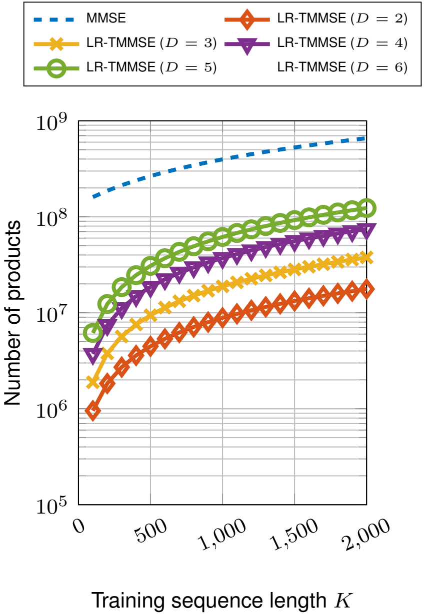

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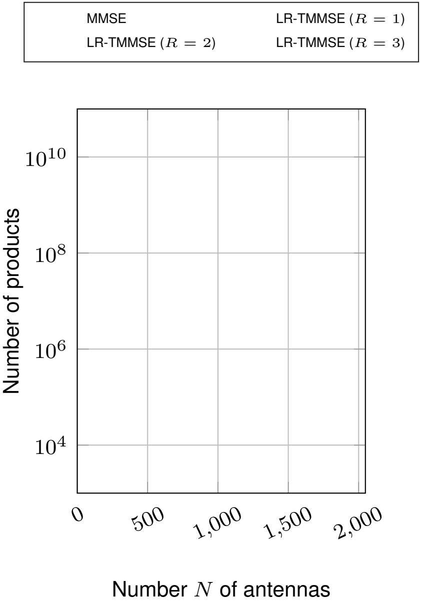

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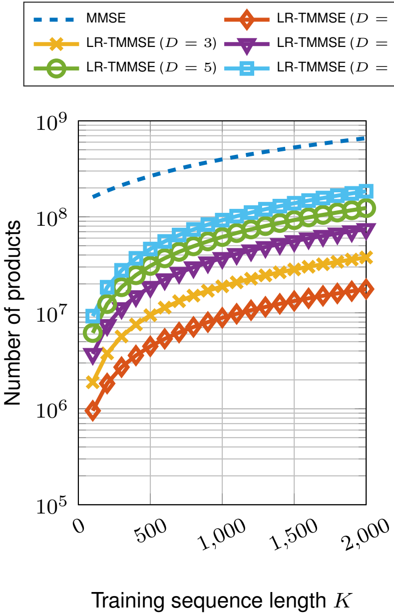

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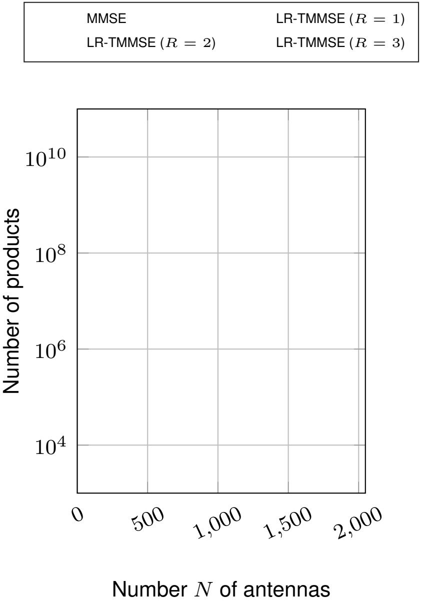

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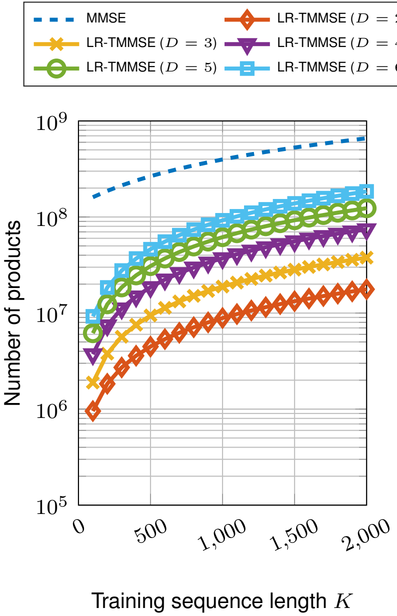

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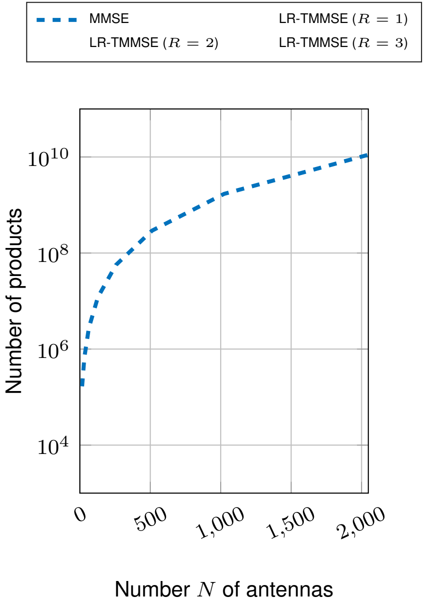

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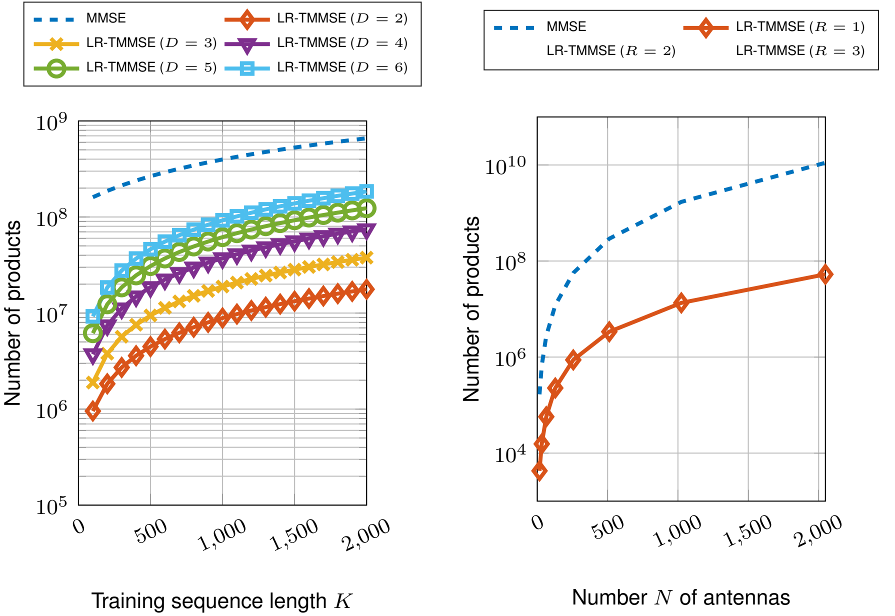

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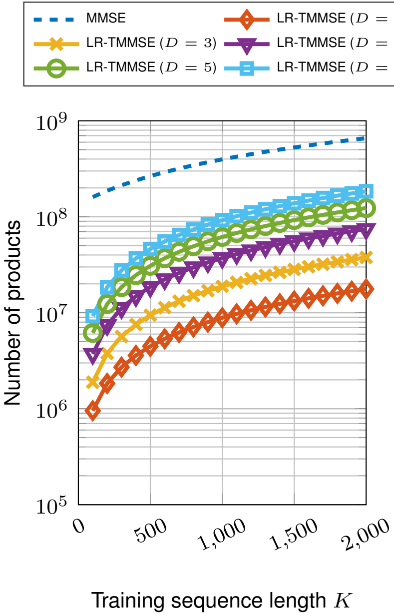

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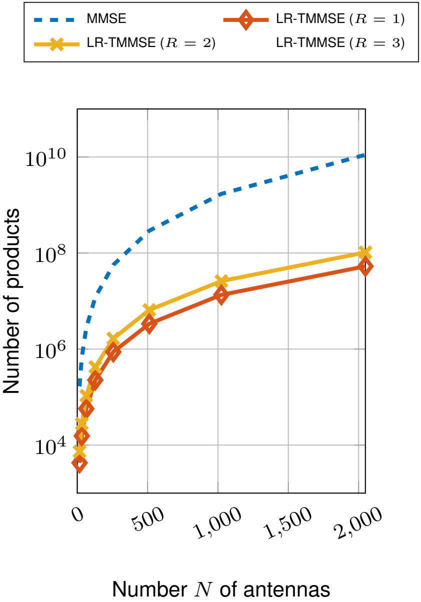

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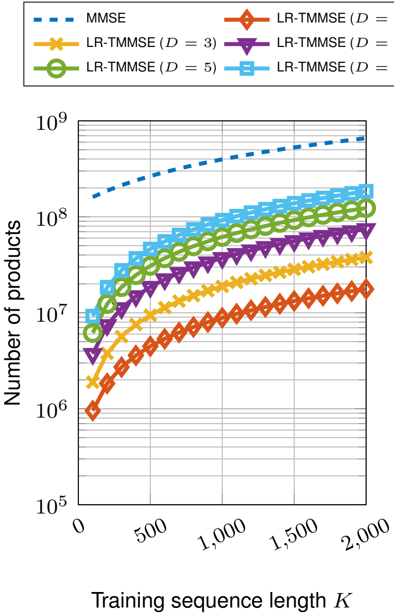

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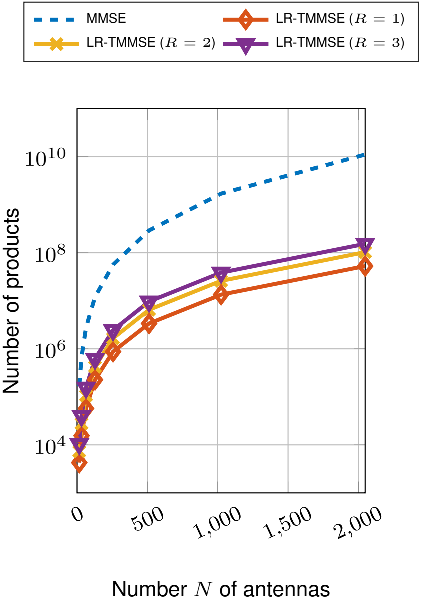

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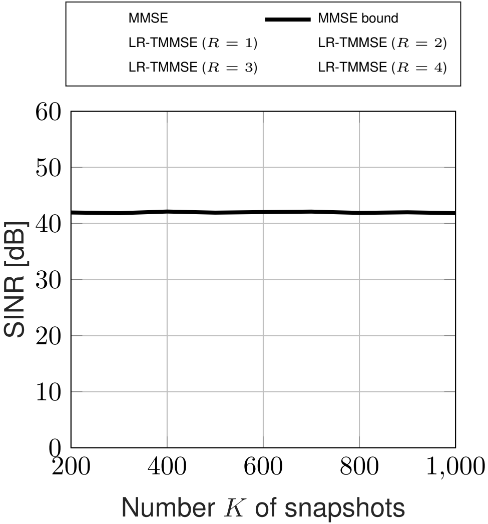
$N = 512$ antennas

SNR = 20 dB

Filter order $D = 3$

$U = 4$ users

$L = 4$ paths



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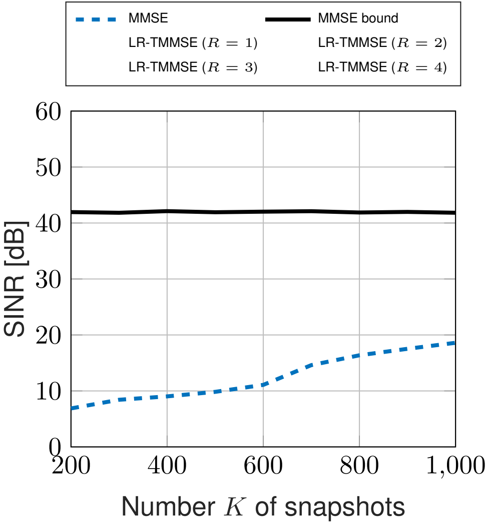

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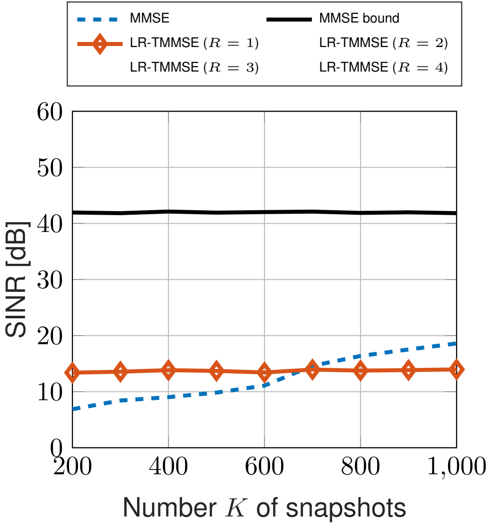

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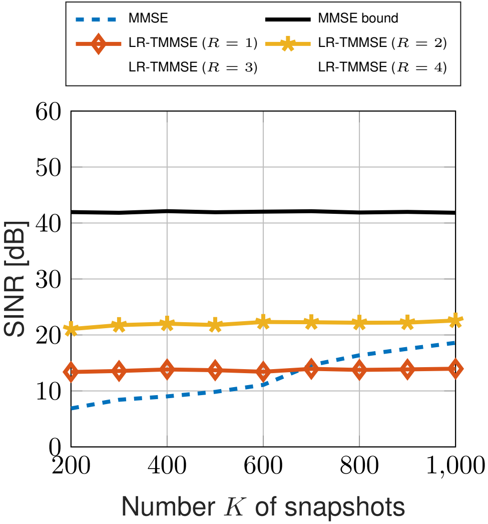

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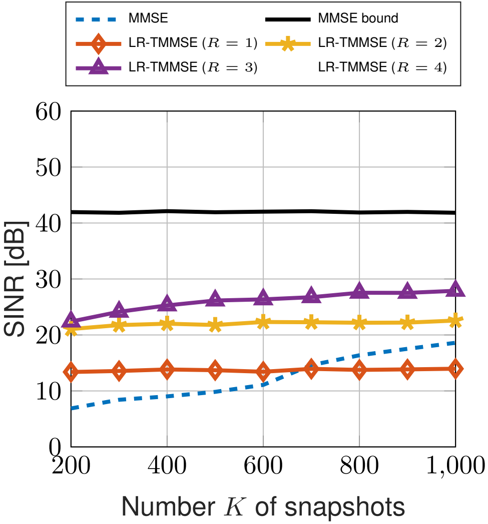
$U = 4$ users

$L = 4$ paths



$$\text{SINR}(\boldsymbol{w}) = \frac{\boldsymbol{w}^{\mathsf{H}} \boldsymbol{R}_{xx} \boldsymbol{w}}{\boldsymbol{w}^{\mathsf{H}} (\boldsymbol{R}_{ii} + \boldsymbol{R}_{bb}) \boldsymbol{w}} \quad (28)$$

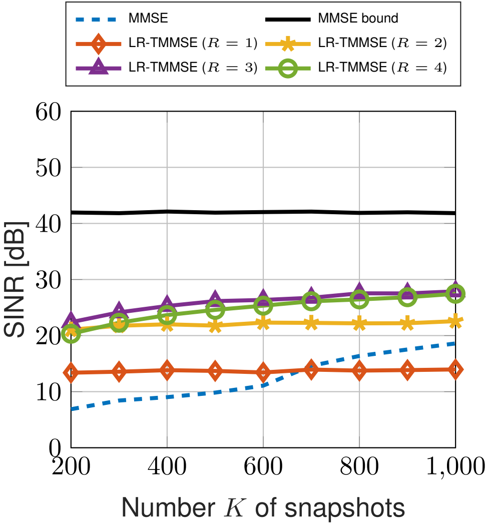
$N = 512$ antennas

SNR = 20 dB

Filter order $D = 3$

$U = 4$ users

$L = 4$ paths

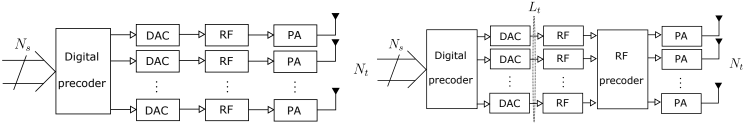


Part II: MmWave Massive MIMO Transceiver Design

Related publications

- *IEEE Journal of Selected Topics in Signal Processing, v. 12, n. 2, p. 298–312, May, 2018;*
- *IEEE Access (under revision)*

Massive MIMO Precoding – Energy Efficiency



(a) Fully-digital (b) Hybrid analog/digital

Large-scale antenna arrays at transmitting side

- Challenges: power consumption, energy efficiency
- Fully digital vs. hybrid A/D (fully- and partially-connected)
- Low-res. data converters → pow. amps. close to saturation (more efficient)

Contributions

- Definition of the quantized hybrid precoding problem
- Assessment of performance losses due to hardware and quantization
- Presentation of novel hybrid A/D precoding techniques

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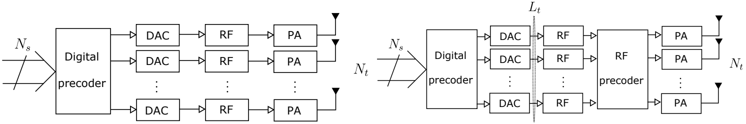

(c) Fully-digital (d) Hybrid analog/digital

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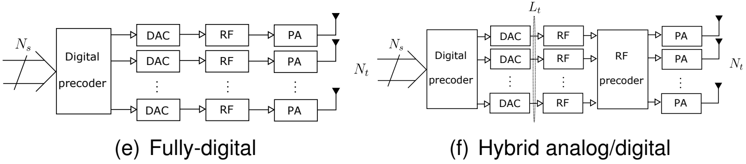

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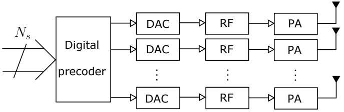
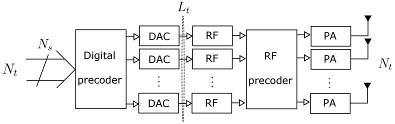


(g) Fully-digital



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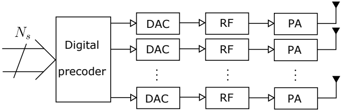
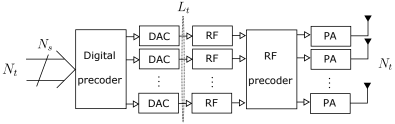


(i) Fully-digital



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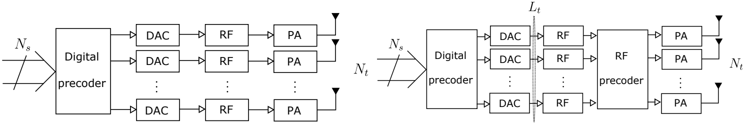

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Signal Model

- Single-user mmWave MIMO system with $N_r \times N_t$ antennas
- Received signal:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \in \mathbb{C}^{N_r} \tag{29}$$

- Transmitted signal with DAC and RF losses:

$$\boldsymbol{x} = \frac{1}{\sqrt{L_{\mathsf{RF}}}} \boldsymbol{F}_{\mathsf{RF}} \mathcal{Q}_b(\boldsymbol{F}_{\mathsf{BB}} \boldsymbol{s}) = \frac{1}{\sqrt{L_{\mathsf{RF}}}} \tilde{\boldsymbol{x}} \in \mathbb{C}^{N_t}, \tag{30}$$

L_{RF} RF losses, L_t TX RF chains, $\boldsymbol{F}_{\mathsf{RF}} \in \mathbb{C}^{N_t \times L_t}$ and $\boldsymbol{F}_{\mathsf{BB}} \in \mathbb{C}^{L_t \times N_s}$ analog and baseband precoders, resp.

- Channel model

$$\boldsymbol{H} = \sqrt{\frac{N_t N_r}{L}} \sum_{\ell=1}^{L} \alpha_\ell \boldsymbol{a}_r\left(\phi_\ell^{(r)}, \theta_\ell^{(r)}\right) \boldsymbol{a}_t\left(\phi_\ell^{(t)}, \theta_\ell^{(t)}\right)^{\mathsf{H}} \in \mathbb{C}^{N_r \times N_t}, \tag{31}$$

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- Proposed quantized signal model

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Quantized Precoding Problem

Problem Formulation

Assuming perfect channel state information (CSI) and whitening:

$$\begin{aligned} \underset{\boldsymbol{F}_{\mathsf{RF}}, \boldsymbol{F}_{\mathsf{BB}}}{\text{maximize}} \quad & \log_2 \det\left(\boldsymbol{I}_{N_r} + \frac{1-\rho_b}{L_{\mathsf{RF}}} \boldsymbol{R}_{n_G n_G}^{-1/2} \boldsymbol{H} \boldsymbol{F}_{\mathsf{RF}} \boldsymbol{F}_{\mathsf{BB}} \boldsymbol{F}_{\mathsf{BB}}^{\mathsf{H}} \boldsymbol{F}_{\mathsf{RF}}^{\mathsf{H}} \boldsymbol{H}^{\mathsf{H}} \boldsymbol{R}_{n_G n_G}^{-1/2,\mathsf{H}}\right) \\ \text{subject to} \quad & [\boldsymbol{F}_{\mathsf{RF}}]_{u,v} \in \mathcal{F}_{\mathsf{RF}}, \forall u \forall v, \mathbb{E}\left[\|\tilde{\boldsymbol{x}}\|_2^2\right] \leq P_{\mathsf{max}}. \end{aligned} \tag{35}$$

where $\tilde{\boldsymbol{x}} = \boldsymbol{F}_{\mathsf{RF}} \boldsymbol{\Upsilon}_b \boldsymbol{u} + \boldsymbol{F}_{\mathsf{RF}} \boldsymbol{e} \in \mathbb{C}^{N_t}$

- It is general to model the (un)quantized hybrid and fully-digital precoding problems
- Sub-optimal solution: optimize $\boldsymbol{F}_{\mathsf{RF}}$ and $\boldsymbol{F}_{\mathsf{BB}}$ independently

Analog Precoder $\boldsymbol{F}_{\mathsf{RF}}$ Design

- Fully-connected: alternating projection method[2]
- Partially-connected: maximum eigenmode transmission by power method

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[2] J. A. Tropp et al, *"Designing structured tight frames via an alternating projection method,"* IEEE Transactions on Information Theory, v. 51, n. 1, p. 188-209, 2005.

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Assuming perfect channel state information (CSI) and whitening:

$$\begin{aligned} \underset{\boldsymbol{F}_{\mathsf{RF}},\,\boldsymbol{F}_{\mathsf{BB}}}{\text{maximize}} \quad & \log_2 \det\left(\boldsymbol{I}_{N_r} + \frac{1-\rho_b}{L_{\mathsf{RF}}} \boldsymbol{R}_{n_G n_G}^{-1/2} \boldsymbol{H} \boldsymbol{F}_{\mathsf{RF}} \boldsymbol{F}_{\mathsf{BB}} \boldsymbol{F}_{\mathsf{BB}}^{\mathsf{H}} \boldsymbol{F}_{\mathsf{RF}}^{\mathsf{H}} \boldsymbol{H}^{\mathsf{H}} \boldsymbol{R}_{n_G n_G}^{-1/2,\mathsf{H}}\right) \\ \text{subject to} \quad & [\boldsymbol{F}_{\mathsf{RF}}]_{u,v} \in \mathcal{F}_{\mathsf{RF}},\ \forall u \forall v,\ \mathbb{E}\left[\|\tilde{\boldsymbol{x}}\|_2^2\right] \leq P_{\mathsf{max}}. \end{aligned} \tag{35}$$

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- It is general to model the (un)quantized hybrid and fully-digital precoding problems
- Sub-optimal solution: optimize $\boldsymbol{F}_{\mathsf{RF}}$ and $\boldsymbol{F}_{\mathsf{BB}}$ independently

Analog Precoder $\boldsymbol{F}_{\mathsf{RF}}$ Design

- Fully-connected: alternating projection method[2]
- Partially-connected: maximum eigenmode transmission by power method

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Baseband Precoder $\boldsymbol{F}_{\mathsf{BB}}$ Design

- Design baseband filter as optimal precoder in infinite-resolution DAC scenarios

$$\begin{aligned} \underset{\boldsymbol{F}_{\mathsf{BB}}}{\text{maximize}} \quad & \log_2 \det\left(\boldsymbol{I}_{N_r} + \boldsymbol{R}_{nn}^{-1}\boldsymbol{H}_{\mathsf{eq}}\boldsymbol{F}_{\mathsf{BB}}\boldsymbol{F}_{\mathsf{BB}}^{\mathsf{H}}\boldsymbol{H}_{\mathsf{eq}}^{\mathsf{H}}\right) \\ \text{subject to} \quad & \mathbb{E}\left[\|\tilde{\boldsymbol{x}}\|_2^2\right] \le P_{\mathsf{max}}. \end{aligned} \tag{36}$$

- Avoids causality problem in total noise covariance matrix
- Consider the SVD of the equivalent channel: $\boldsymbol{H}_{\mathsf{eq}} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathsf{H}}$
- SVD precoding + waterfilling power allocation:

$$\boldsymbol{F}_{\mathsf{BB}} = \frac{\sqrt{P_{\mathsf{max}}}}{\|\boldsymbol{F}_{\mathsf{RF}}\boldsymbol{Q}\|_{\mathrm{F}}}\boldsymbol{Q} \tag{37}$$

$$\boldsymbol{Q} = \boldsymbol{V}\boldsymbol{\Lambda}^{1/2} \in \mathbb{C}^{M\times N_s} \tag{38}$$

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Power Consumption and Loss Models

Power Consumption Formulas

- Fully-digital: $P_{\text{D}} = P_{\text{LO}} + P_{\text{PA}} + N_t[2P_{\text{DAC}}(b_{\text{DAC}}, F_s) + P_{\text{RF}}]$
- Hybrid A/D fully-connected:
  $P_{\text{FPSN}} = P_{\text{LO}} + P_{\text{PA}} + L_t[2P_{\text{DAC}}(b_{\text{DAC}}, F_s) + P_{\text{RF}}] + N_t L_t P_{\text{PS}}(b_{\text{PS}})$
- Hybrid A/D partially-connected:
  $P_{\text{PPSN}} = P_{\text{LO}} + P_{\text{PA}} + L_t[2P_{\text{DAC}}(b_{\text{DAC}}, F_s) + P_{\text{RF}}] + N_a L_t P_{\text{PS}}(b_{\text{PS}})$
- Power amplifier: $P_{\text{PA}} = P_x/\eta$, for efficiency η and

$$P_x = \frac{1}{L_{\text{RF}}}\left[(1-\rho_b)\|\boldsymbol{F}_{\text{RF}}\boldsymbol{F}_{\text{BB}}\|_F^2 + \text{Tr}(\boldsymbol{F}_{\text{RF}}\boldsymbol{R}_{ee}\boldsymbol{F}_{\text{RF}}^{\text{H}})\right]$$

RF Devices Loss

- 2-way pow. div: $L_{\text{D}}(N_t)$
- 2-way pow. comb: $L_{\text{C}}(L_t)$
- Phase-shifter (passive or active): L_{PS}

Phase-Shifting Network Loss

- $L_{\text{RF}}^{\text{FPSN}} = L_{\text{D}}(N_t)L_{\text{PS}}L_{\text{C}}(L_t)$.
- $L_{\text{RF}}^{\text{PPSN}} = L_{\text{D}}(N_a)L_{\text{PS}}$.

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- $L_{\mathsf{RF}}^{\mathsf{PPSN}} = L_{\mathsf{D}}(N_a)L_{\mathsf{PS}}$.

Simulation Setup

- $N_t = 64$ and $N_r = 4$ antennas
- $L_t = 4$ RF chains
- $N_s = 4$ data streams
- $L = 5$ channel paths
- $P_{\text{max}} = 1$ W
- Phase shifter resolution: 5 bits
- DAC sampling frequency $F_s = 1$ GHz
- Energy efficiency:

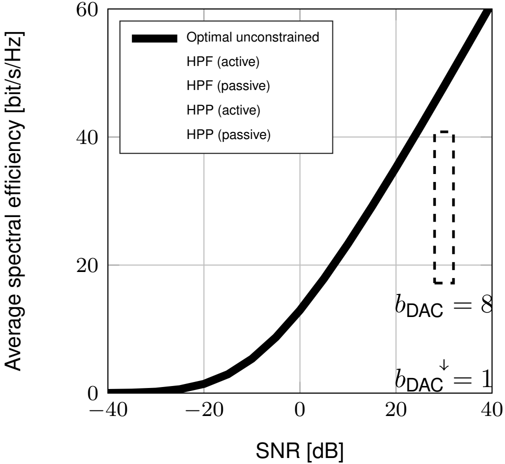
$$\frac{\text{spectral efficiency}}{\text{power consumption}} \quad [\text{bit/J}]$$

Phase shifter implementation

- Active: ↑ power consumption ↓ insertion loss
- Passive: ↓ power consumption ↑ insertion loss

Notation	Value
P_{PA}	$P_x/\eta,\ \eta = 27\%$
P_{PS}	21.6 ; 0 mW
P_{LO}	22.5 mW
P_{H}	3 mW
P_{M}	0.3 mW
P_{LP}	14 mW
P_{RF}	31.6 mW
$\bar{L}_{\text{D}}$	0.6 dB
$\bar{L}_{\text{C}}$	0.6 dB $+ 3$ dB
L_{PS}	-2.3 ; 8.8 dB

Simulation Results



Considering RF hardware losses

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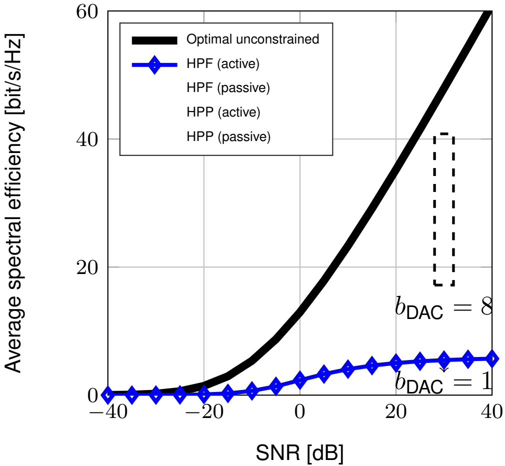

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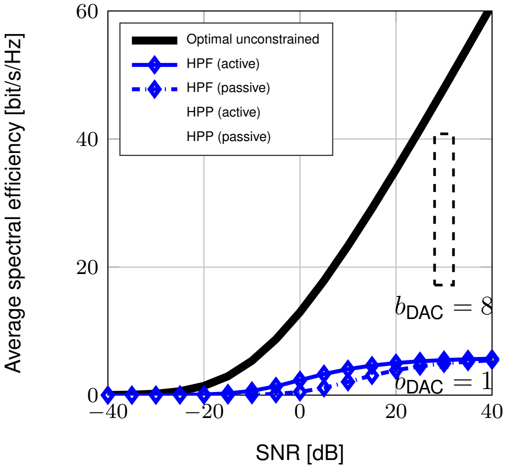

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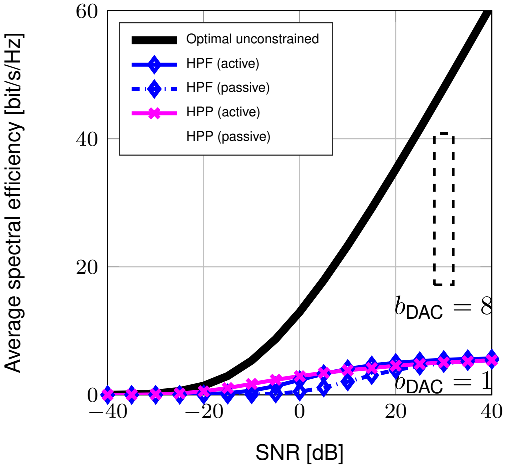

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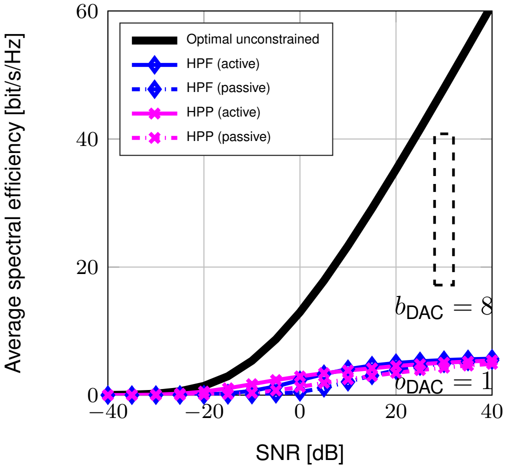

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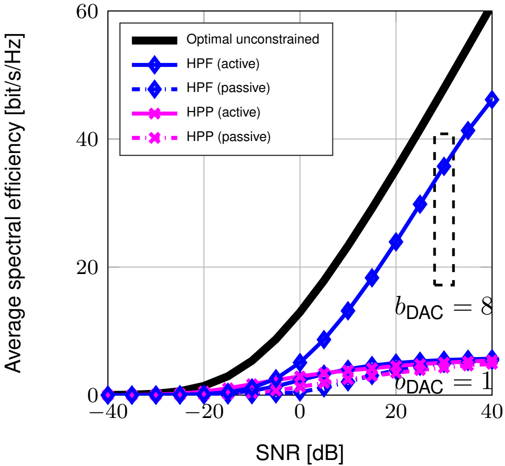

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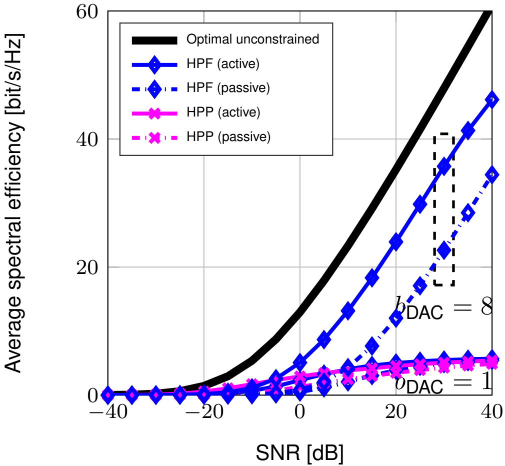

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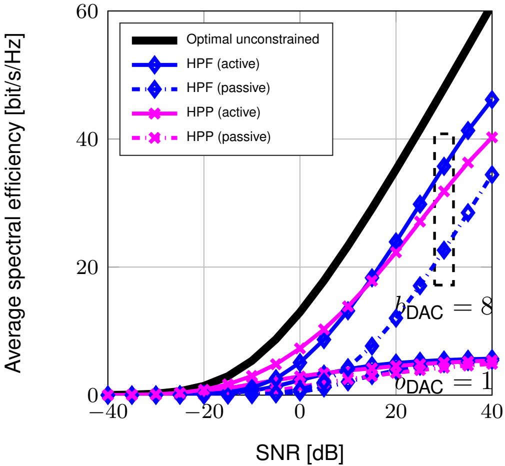

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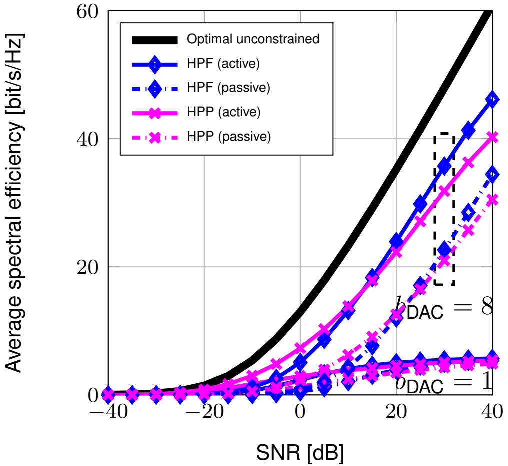

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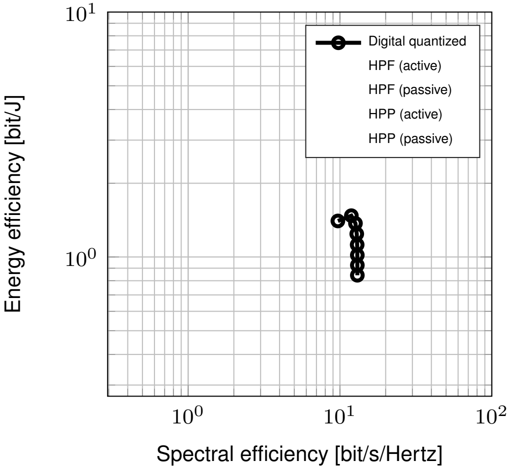

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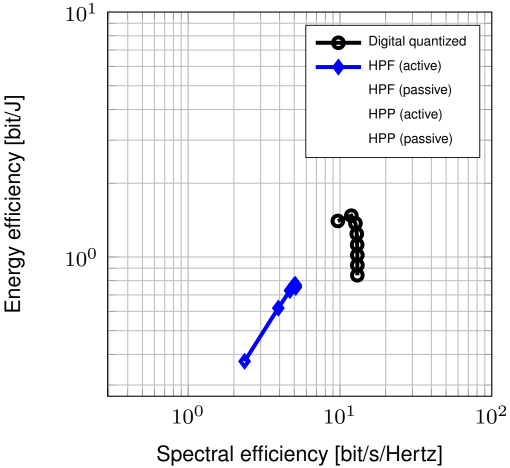

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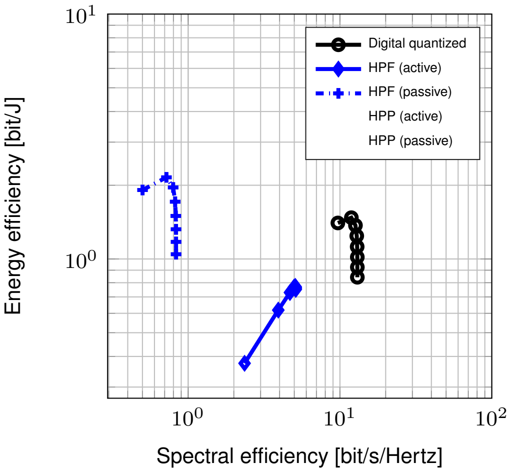

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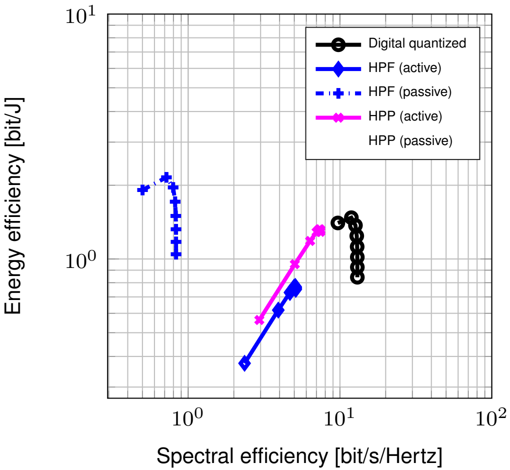

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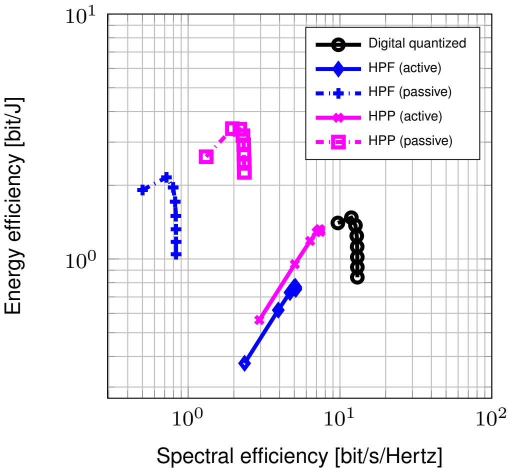

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Massive MIMO only at transmitter (base station)

Double-Sided Massive MIMO

Transceiver Design

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  - Potentially better performance than canonical massive MIMO
  - Wireless backhauling, terahertz communications, among others
- Contributions
  - Low-complexity transceiver schemes with practical CSI requirements
  - Performance evaluation under different propagation conditions

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System Model

Signal and Channel Models

Downlink operation, 1 BS (N_t antennas), U UEs (N_r antennas, each)

$$\boldsymbol{y}_u = \boldsymbol{W}_u^{\mathsf{H}} \boldsymbol{H}_u \boldsymbol{F}_u \boldsymbol{s}_u + \sum_{\substack{j=1 \\ j \neq u}}^{U} \boldsymbol{W}_u^{\mathsf{H}} \boldsymbol{H}_u \boldsymbol{F}_j \boldsymbol{s}_j + \boldsymbol{W}_u^{\mathsf{H}} \boldsymbol{b}_u \in \mathbb{C}^{N_s}, \tag{39}$$

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Multi-Layer Filtering

Two layers: outer and inner layers

- $\boldsymbol{F}_u = \gamma_u \boldsymbol{F}_{\mathsf{o},u} \boldsymbol{F}_{\mathsf{i},u}$, $\boldsymbol{F}_{\mathsf{o},u} \in \mathbb{C}^{N_t \times M_t}$ and $\gamma_u \boldsymbol{F}_{\mathsf{i},u} \in \mathbb{C}^{M_t \times N_s}$
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Each layer, one objective:

- Outer layer: increase SNR
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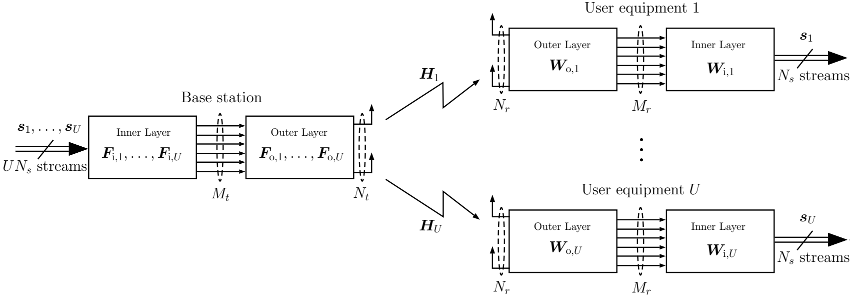

Signal Model (inner filters and effective channels)

Form low-dimensional effective channels!

$$\boldsymbol{H}_{\mathsf{eff},u,j} = \boldsymbol{W}_{\mathsf{o},u}^{\mathsf{H}} \boldsymbol{H}_u \boldsymbol{F}_{\mathsf{o},j} \in \mathbb{C}^{M_r \times M_t}, \quad \gamma_u = \frac{\sqrt{P_t/U}}{\|\boldsymbol{F}_{\mathsf{o},u} \boldsymbol{F}_{\mathsf{i},u}\|_{\mathrm{F}}} \tag{41}$$

$$\boldsymbol{y}_u = \gamma_u \boldsymbol{W}_{\mathsf{i},u}^{\mathsf{H}} \boldsymbol{H}_{\mathsf{eff},u} \boldsymbol{F}_{\mathsf{i},u} \boldsymbol{s}_u + \sum_{\substack{j=1 \\ j \neq u}}^{U} \gamma_j \boldsymbol{W}_{\mathsf{i},u}^{\mathsf{H}} \boldsymbol{H}_{\mathsf{eff},u,j} \boldsymbol{F}_{\mathsf{i},j} \boldsymbol{s}_j + \boldsymbol{W}_{\mathsf{i},u}^{\mathsf{H}} \boldsymbol{b}_{\mathsf{eff},u} \tag{42}$$

CSI Acquisition

First Stage: Outer Layer

CSI necessary for outer layer design

- Statistical CSI (uplink and downlink cov. matrices); or
- Partial CSI: path power and angles

Depend only on macroscopic channel parameters!

Second Stage: Inner Layer

- Estimate low-dimensional effective channels $H_{\text{eff},u,j}$
- Example: classical MMSE estimators

Time Scales

- Macroscopic: update outer layers
- Microscopic: update inner layers **(low complexity!)**

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  - Power-dominant path selection (PPS)
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Simulation Results – Setup

- Investigate multiplexing capabilities
- Achievable sum rate as figure of merit
- Channel conditions
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  - Rich: $L = 64$ rays
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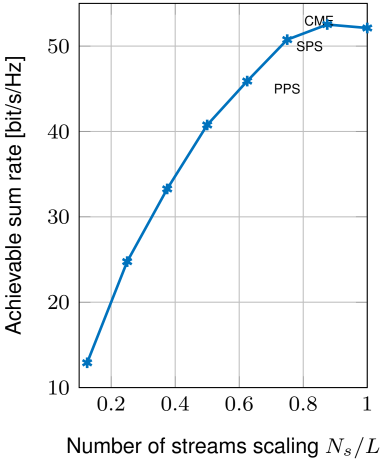

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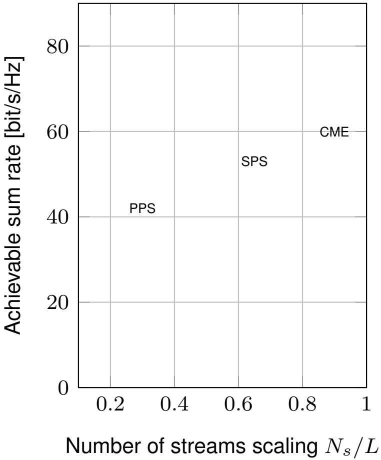

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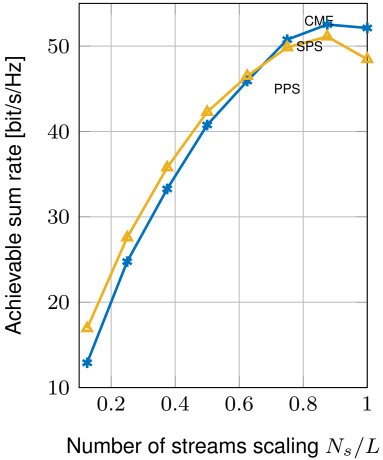

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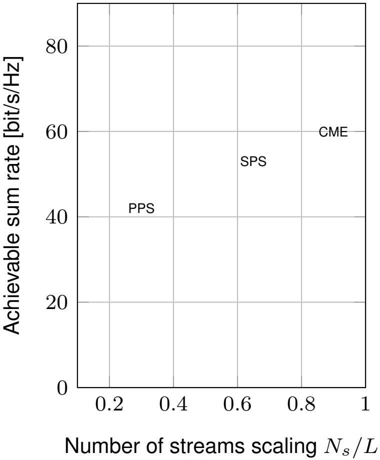

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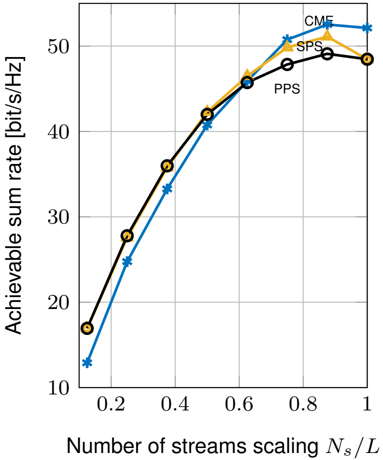

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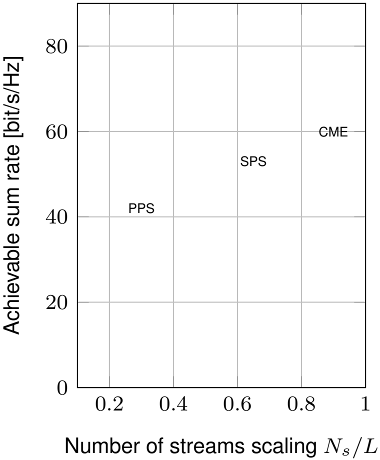

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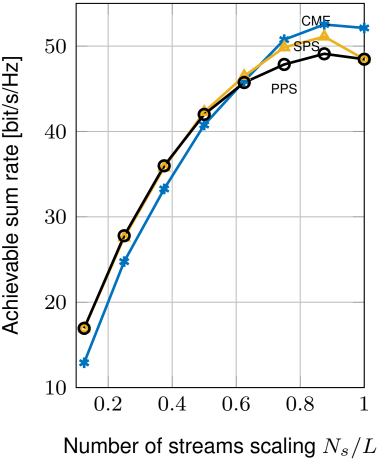

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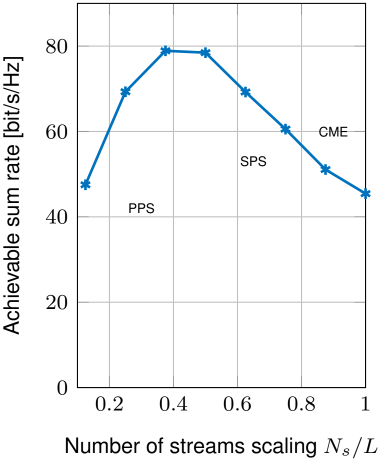

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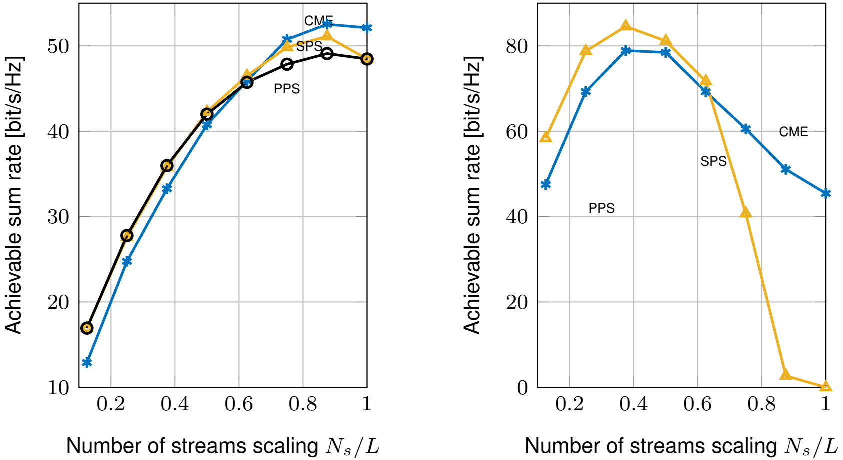

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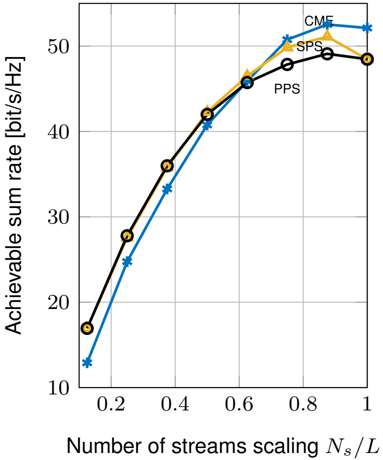

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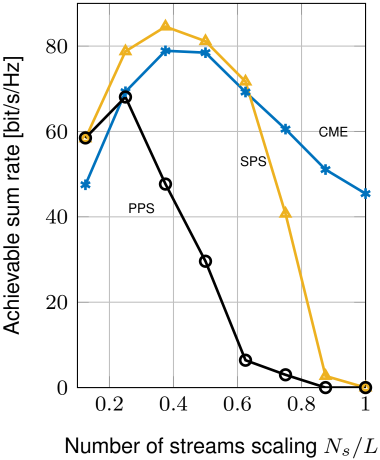

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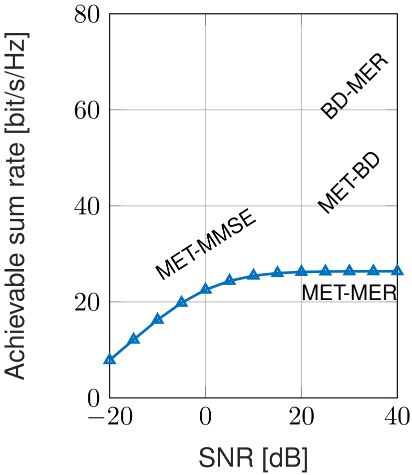
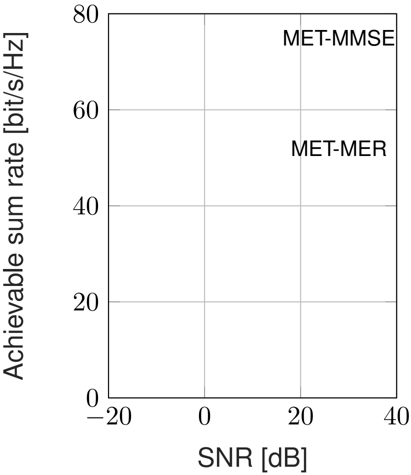


$U = 4$ users, $M_t = M_r = 4$



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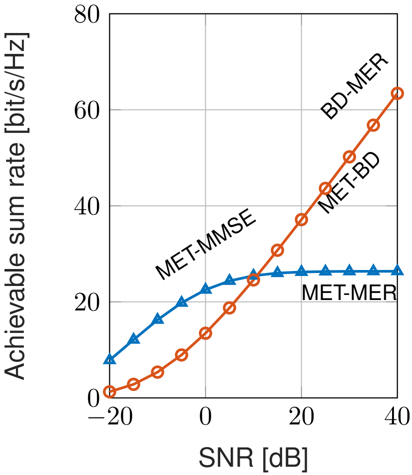

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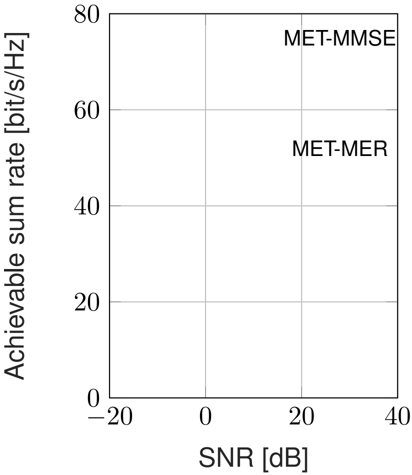

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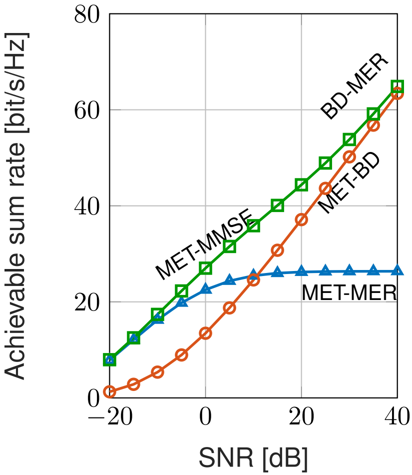

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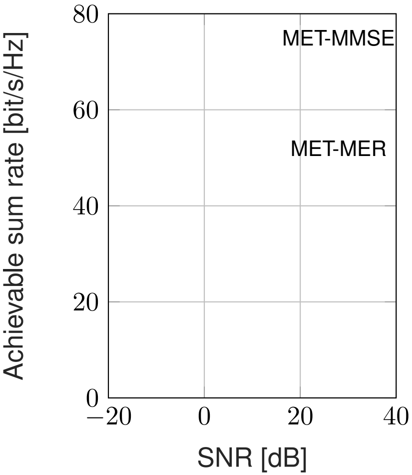

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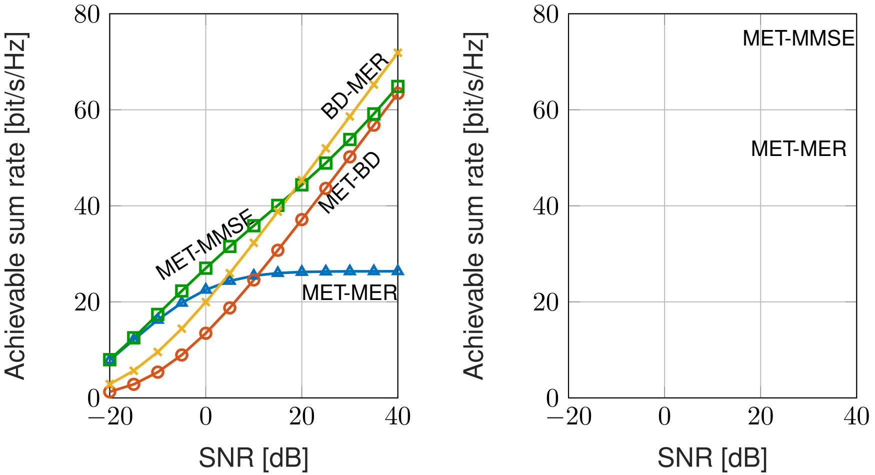

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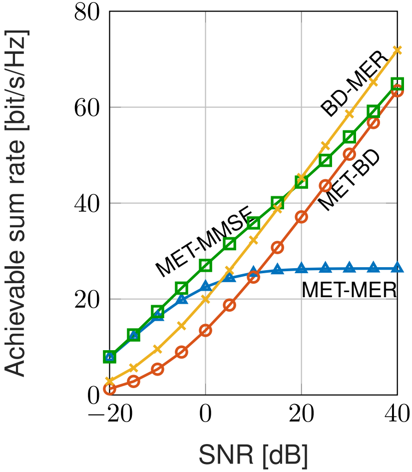

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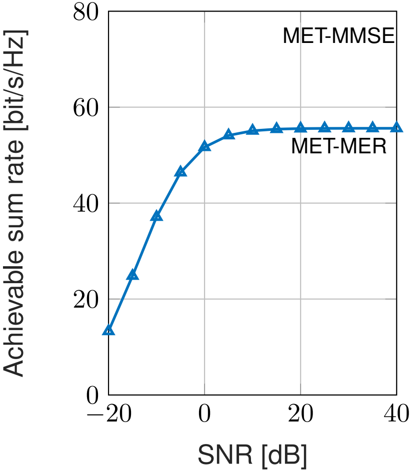

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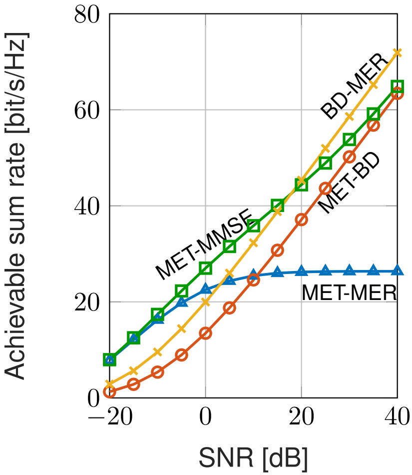

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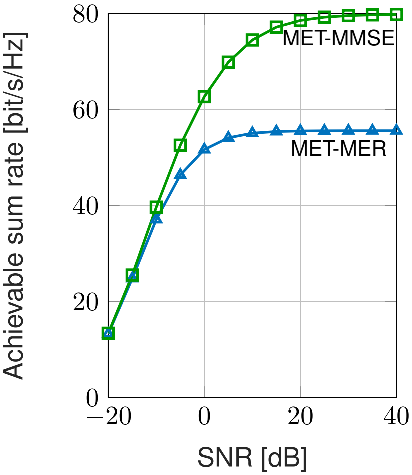

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Part III: MmWave Channel Estimation with Synchronization Impairments

Related publications

- *Proc. IEEE ICASSP 2019*
- *Wideband extension under preparation*

MmWave Channel Estimation with Synchronization Impairments

- High-quality oscillators in mmWave are expensive
- Carrier signal far from ideal
- Impairments:
  - Carrier frequency offset (CFO)
  - Phase noise (PN)
- Classical approach: compensate impairments prior to beamforming and channel estimation
- **MmWave:** low SNR operation $\rightarrow$ classical methods may fail[4]
- Joint wideband mmWave channel parameters, PN and CFO estimation

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System Model

Time-Domain Protocol

- Sample period T_o
- Symbol period T_s: comprises N_o samples $\rightarrow T_s = N_o T_o$
- Block period T_b: comprises N_s symbols $\rightarrow T_b = N_s T_s$
- Frame period T_s: comprises N_b blocks $\rightarrow T_f = N_b T_b$

System Parameters

- $(N_t \times N_r)$ single-user MIMO system
- Transmission of N_p-length pilot sequences
- Transmit and receive codebooks of length M_t and M_r, respectively
- Single local oscillator at each end: Ω [rad/s]
- Phase Noise: $\phi_n = \phi_{n-1} + w_n$ (Wiener process)

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Channel matrix at frame n_f and tap n_c

$$\boldsymbol{H}_{n_f,n_c} = \sqrt{\frac{N_t N_r}{L}} \sum_{\ell=1}^{L} \alpha_{n_f,\ell} g_{n_c,\ell} \boldsymbol{a}_r \left(\phi_\ell^{(r)}, \theta_\ell^{(r)}\right) \boldsymbol{a}_t^{\mathsf{T}} \left(\phi_\ell^{(t)}, \theta_\ell^{(t)}\right) \in \mathbb{C}^{N_r \times N_t}$$

- $\alpha_{n_f,\ell}$ – frame-variant complex channel path gain
- $g_{n_c,\ell} = g(n_c T_s - \tau_\ell)$ – effective pulse shaping function

Parameters Time-Scale

- PN: Sample scale – ϕ_{n_o}, $n_o = 1, \ldots, N_o$
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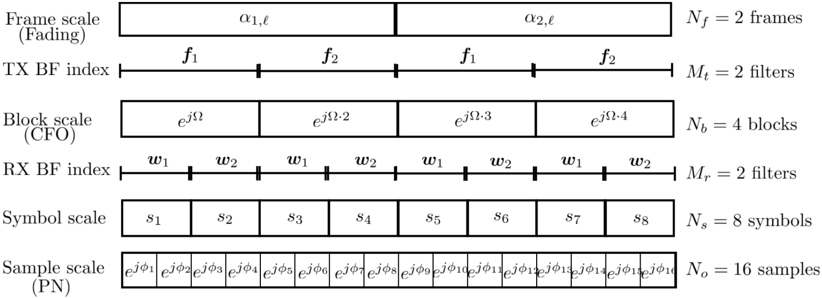
Received signal at sample n_o, symbol n_s, block n_b, frame n_f, filtered by transmit beamformer $\boldsymbol{f}_{m_t}$ and receive beamformer $\boldsymbol{w}_{m_r}$:

$$y_{m_r,m_t,n_o,n_s,n_b,n_f} = e^{\jmath(\Omega \cdot n_b + \phi_{n_o})} \sum_{n_c=0}^{N_c-1} \boldsymbol{w}_{m_r}^{\mathsf{H}} \boldsymbol{H}_{n_f,n_c} \boldsymbol{f}_{m_t} s_{n_s-n_c} + \boldsymbol{w}_{m_r}^{\mathsf{H}} \boldsymbol{b}_{m_t,n_b,n_f,n_s,n_o}$$



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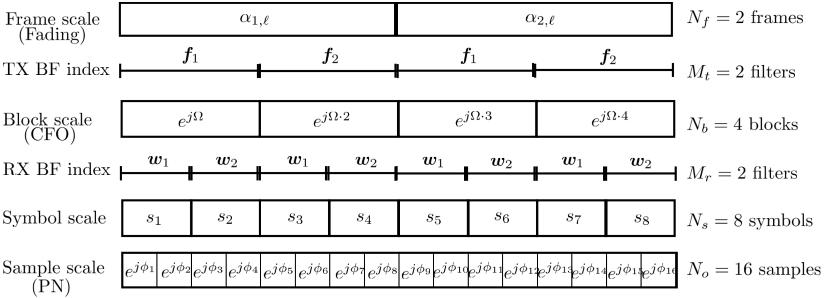

System Model – Tensor Formulation

Effective channel tensor

$$\mathcal{C} = \mathcal{I}_{6,L} \times_1 \boldsymbol{A}_r \times_2 \boldsymbol{A}_t^* \times_3 \boldsymbol{G} \times_4 \boldsymbol{\Phi} \times_5 \boldsymbol{\Omega} \times_6 \boldsymbol{\Gamma} \tag{43}$$

- $\boldsymbol{A}_r \in \mathbb{C}^{N_r \times L}$ and $\boldsymbol{A}_t \in \mathbb{C}^{N_t \times L}$ – spatial signatures
- $\boldsymbol{G} \in \mathbb{C}^{N_c \times L}$ – time signature
- $\boldsymbol{\Phi} = \frac{1}{\sqrt{L}} \operatorname{Diag}(e^{\jmath\phi_1}, \dots e^{\jmath\phi_{N_o}}) \mathbf{1}_{N_o \times L} \in \mathbb{C}^{N_o \times L}$ – PN matrix **(rank-1)**
- $\boldsymbol{\Omega} = \frac{1}{\sqrt{L}} \operatorname{Diag}(e^{\jmath\Omega}, \dots, e^{\jmath\Omega \cdot N_f}) \mathbf{1}_{N_b \times L} \in \mathbb{C}^{N_b \times L}$ – CFO matrix
- $\boldsymbol{\Gamma} \in \mathbb{C}^{N_f \times L}$ – fading matrix

Received signal tensor

$$\mathcal{Y} = \mathcal{C} \times_1 \boldsymbol{W}^{\mathsf{H}} \times_2 \boldsymbol{F}^{\mathsf{T}} \times_3 \boldsymbol{S}^{\mathsf{T}} + \mathcal{Z} \tag{44}$$

$$= \mathcal{I}_{6,L} \times_1 \boldsymbol{W}^{\mathsf{H}}\boldsymbol{A}_r \times_2 \boldsymbol{F}^{\mathsf{T}}\boldsymbol{A}_t^* \times_3 \boldsymbol{S}^{\mathsf{T}}\boldsymbol{G} \times_4 \boldsymbol{\Phi} \times_5 \boldsymbol{\Omega} \times_6 \boldsymbol{\Gamma} + \mathcal{Z}. \tag{45}$$

Canonical polyadic decomposition (CPD) model!

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- $\boldsymbol{A}_r \in \mathbb{C}^{N_r \times L}$ and $\boldsymbol{A}_t \in \mathbb{C}^{N_t \times L}$ – spatial signatures
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- $\boldsymbol{\Gamma} \in \mathbb{C}^{N_f \times L}$ – fading matrix

Received signal tensor

$$\mathcal{Y} = \mathcal{C} \times_1 \boldsymbol{W}^{\mathsf{H}} \times_2 \boldsymbol{F}^{\mathsf{T}} \times_3 \boldsymbol{S}^{\mathsf{T}} + \mathcal{Z} \tag{44}$$

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Canonical polyadic decomposition (CPD) model!

System Model – Tensor Formulation

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Parameter Estimation

$$\mathcal{Y} = \mathcal{I}_{6,L} \times_1 \boldsymbol{W}^{\mathsf{H}} \boldsymbol{A}_r \times_2 \boldsymbol{F}^{\mathsf{T}} \boldsymbol{A}_t^* \times_3 \boldsymbol{S}^{\mathsf{T}} \boldsymbol{G} \times_4 \boldsymbol{\Phi} \times_5 \boldsymbol{\Omega} \times_6 \boldsymbol{\Gamma} + \mathcal{Z}$$

Steps

1. Factorize received signal tensor $\mathcal{Y}$ into CPD model
2. Solve permutation ambiguity
3. Estimate the path angles and delays by solving sparse recovery problems

$$\begin{aligned} &\underset{\boldsymbol{v}_r}{\text{minimize}} && \|\boldsymbol{v}_r\|_1 \\ &\text{subject to} && \|\boldsymbol{q}_{(1)} - [\boldsymbol{I}_L \otimes (\boldsymbol{W}^{\mathsf{H}} \boldsymbol{\Psi}_r)]\boldsymbol{v}_r\|_2 \leq \sigma, \end{aligned} \tag{46}$$

$$\begin{aligned} &\underset{\boldsymbol{v}_t}{\text{minimize}} && \|\boldsymbol{v}_t\|_1 \\ &\text{subject to} && \|\boldsymbol{q}_{(2)} - [\boldsymbol{I}_L \otimes (\boldsymbol{F}^{\mathsf{T}} \boldsymbol{\Psi}_t)]\boldsymbol{v}_t\|_2 \leq \sigma, \end{aligned} \tag{47}$$

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4. Estimate PN and CFO directly from the CPD factors
5. Estimate channel fading matrix as

$$\hat{\boldsymbol{\Gamma}} = \boldsymbol{Y}_{(6)} \left\{ \left[\boldsymbol{\Omega} \diamond \boldsymbol{\Phi} \diamond (\boldsymbol{S}^{\mathsf{T}} \boldsymbol{G}) \diamond (\boldsymbol{F}^{\mathsf{T}} \boldsymbol{A}_t^*) \diamond (\boldsymbol{W}^{\mathsf{H}} \boldsymbol{A}_r) \right]^{\mathsf{T}} \right\}^{\dagger}. \tag{49}$$

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Simulation Results

Figures of Merit

Angles, delays and CFO $(L = 1)$

$$\text{NMSE}(x) = \sum_{\ell=1}^{L} \frac{|x_\ell - \hat{x}_\ell|^2}{|x_\ell|^2} \tag{50}$$

Phase noise

$$\text{NMSE}(\phi) = \sum_{n_o=1}^{N_o} \frac{|\phi_{n_o} - \hat{\phi}_{n_o}|^2}{|\phi_{n_o}|^2} \tag{51}$$

Fading matrix

$$\text{NMSE}(\mathbf{\Gamma}) = \frac{\|\mathbf{\Gamma} - \hat{\mathbf{\Gamma}}\|_{\text{F}}^2}{\|\mathbf{\Gamma}\|_{\text{F}}^2} \tag{52}$$

Calculate NMSE for different codebook lengths and samples number N_o

Parameter Setup

- $N_t = N_r = 16$ antennas
- $N_s = N_b = N_f = 2$
- Sampling period $T_s = 0.1\ \mu s$
- Carrier frequency 28 GHz
- 10 ppm CFO: 280 kHz
- 2000 independent trials

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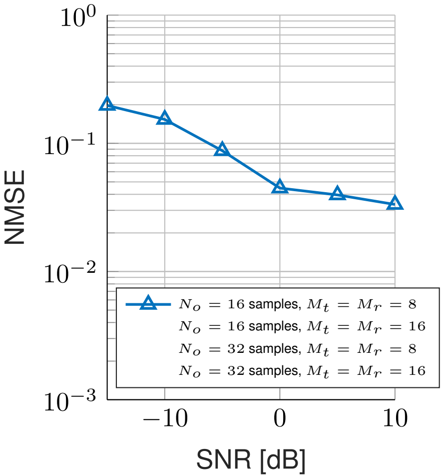
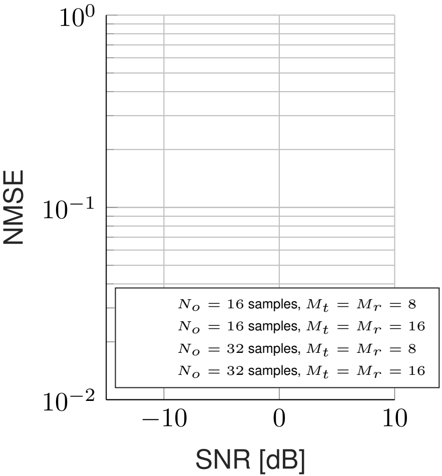
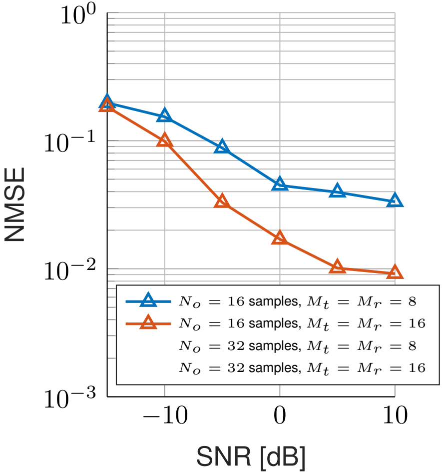


Angles of arrival



Path delays

Simulation Results



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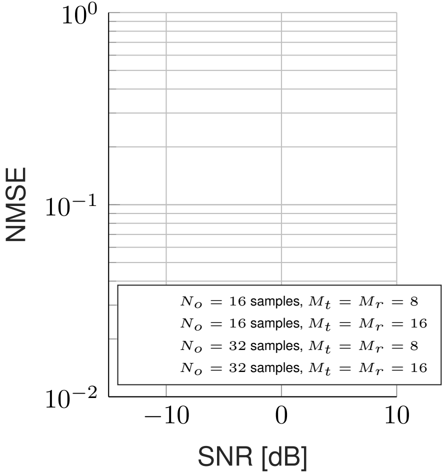

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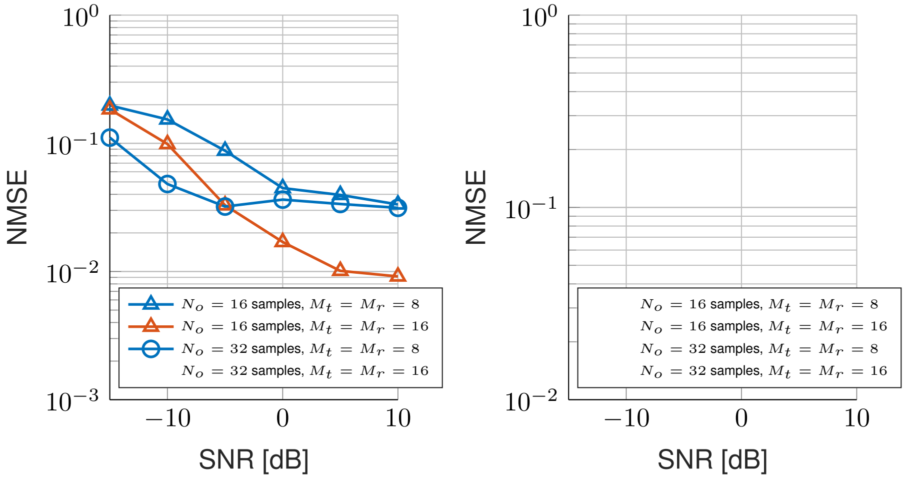

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Path delays

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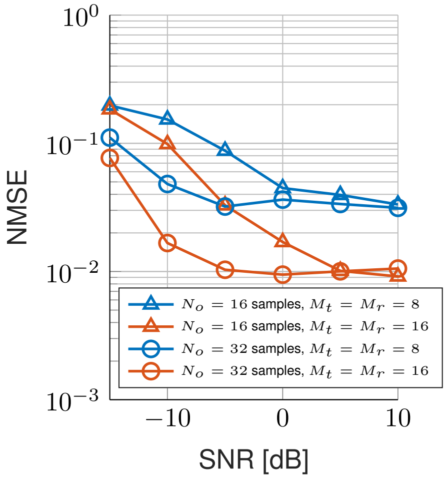

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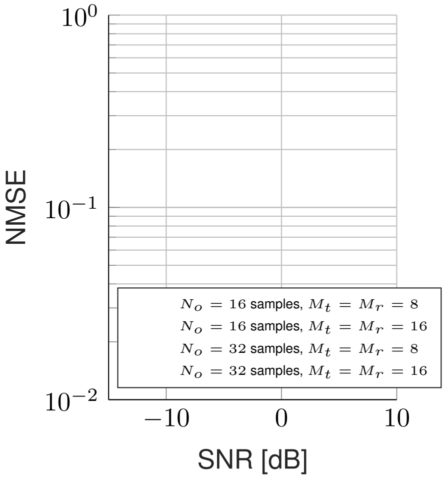

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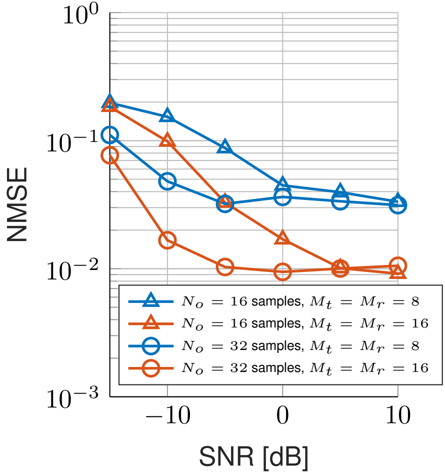

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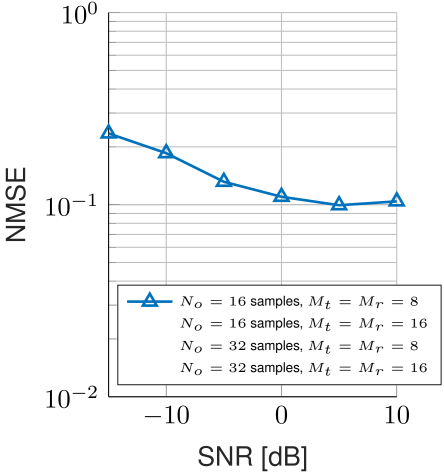

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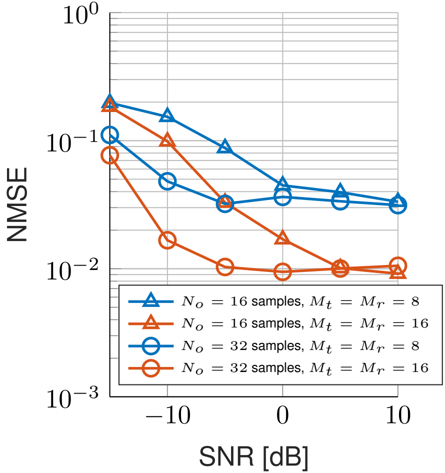

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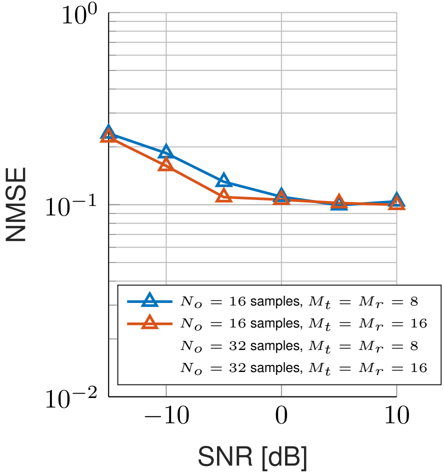

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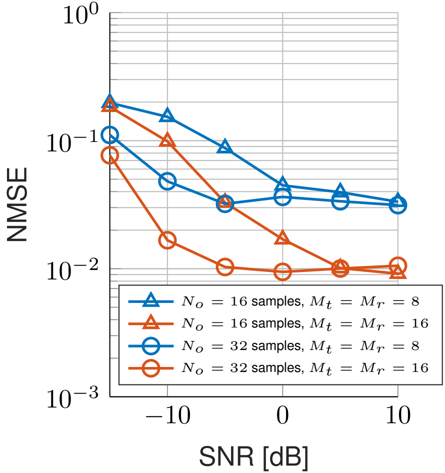

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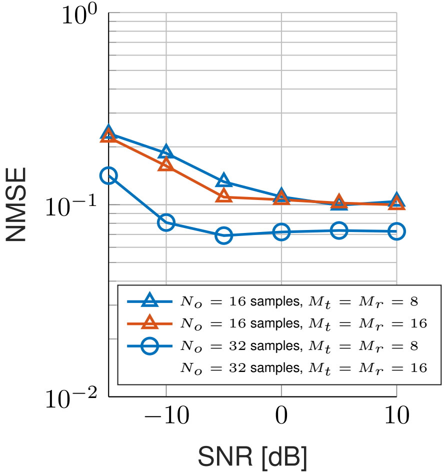

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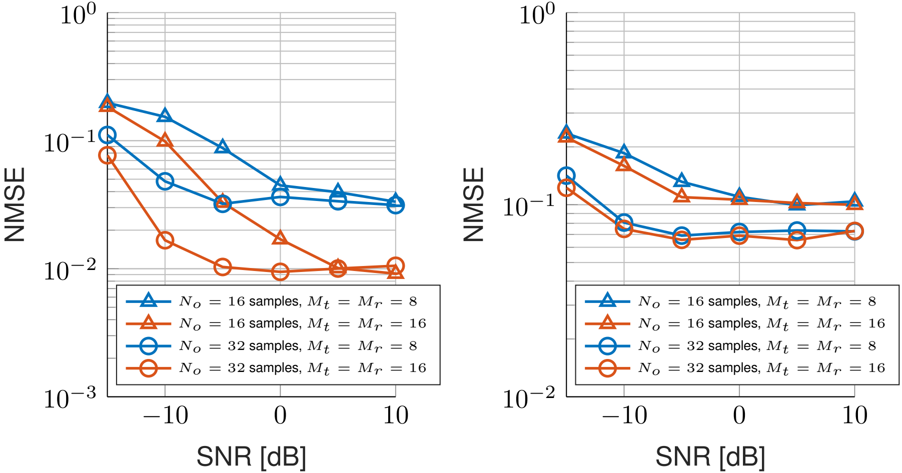
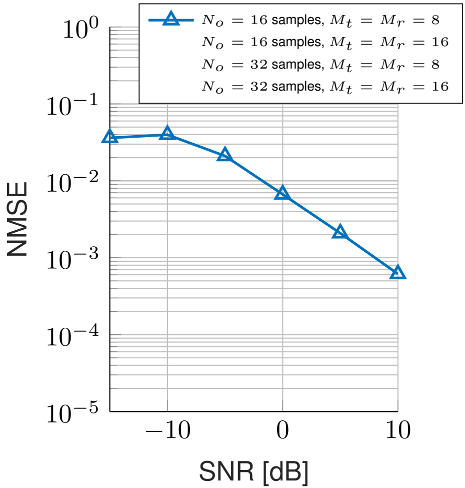


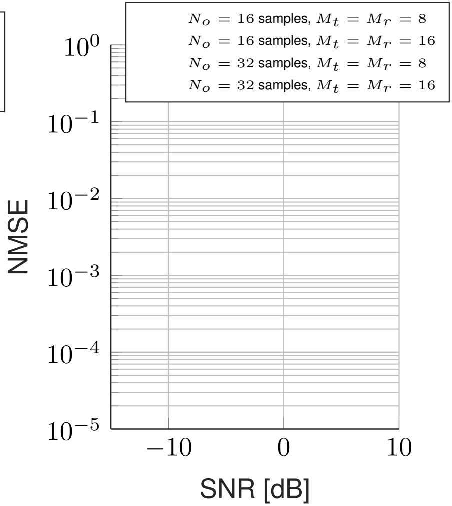
Angles of arrival

Path delays

Simulation Results



Phase noise



CFO

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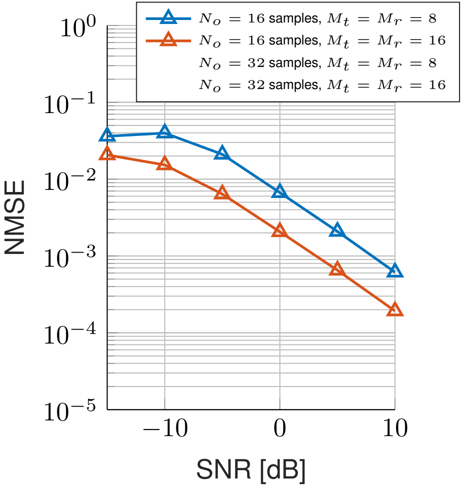
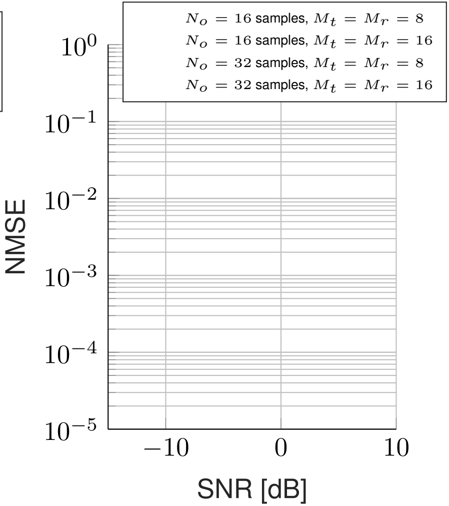


Phase noise



CFO

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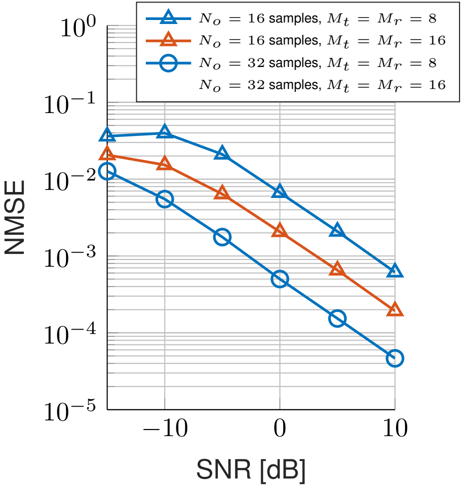

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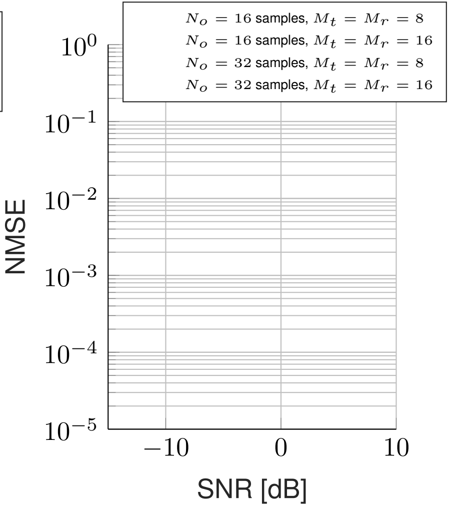

CFO

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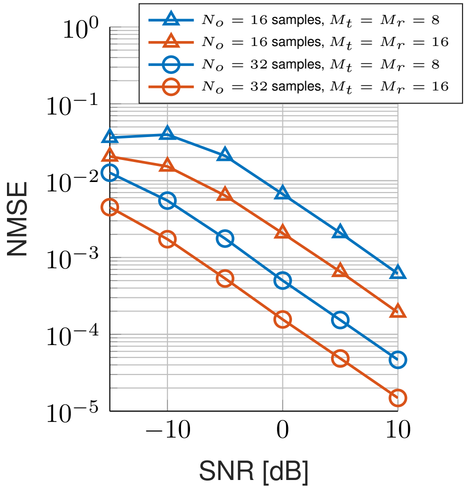
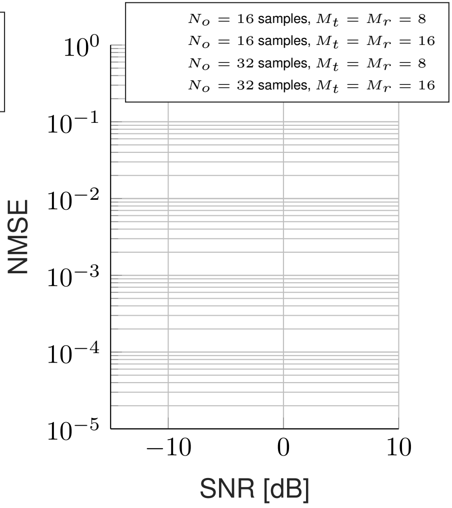


Phase noise



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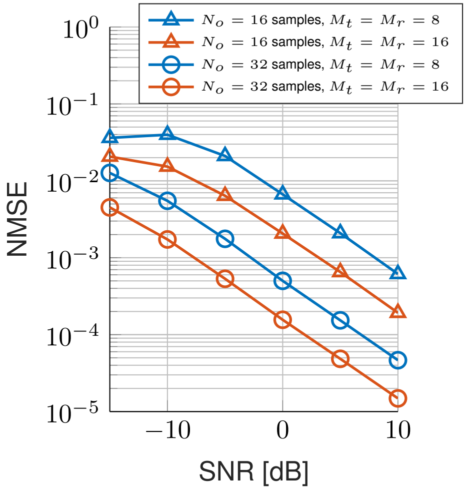
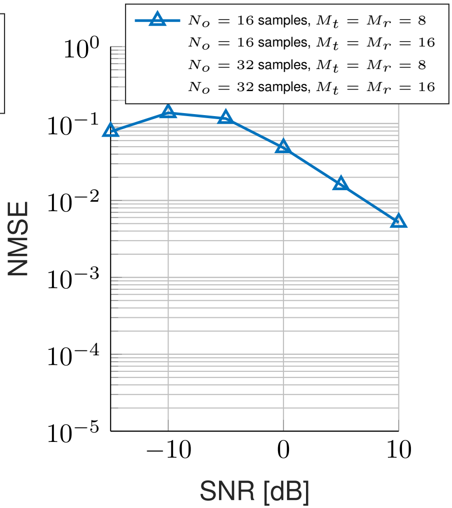


Phase noise



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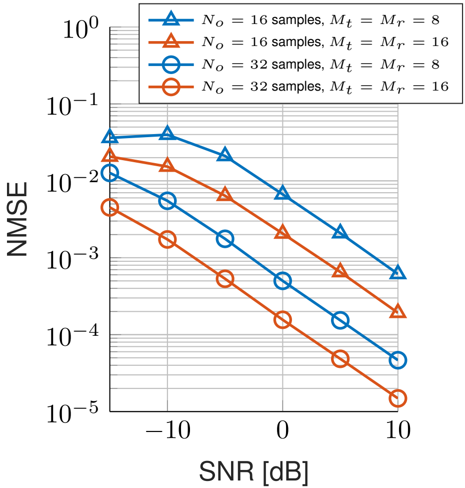
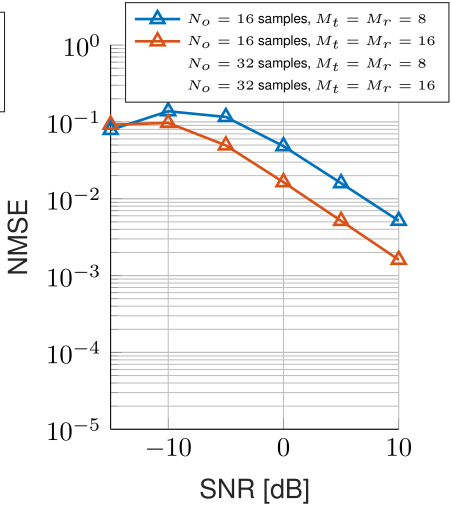


Phase noise



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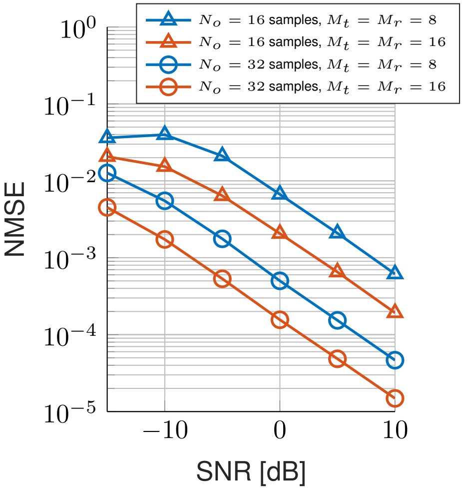
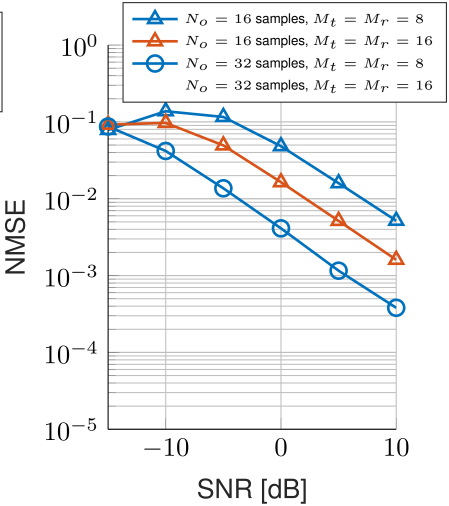


Phase noise



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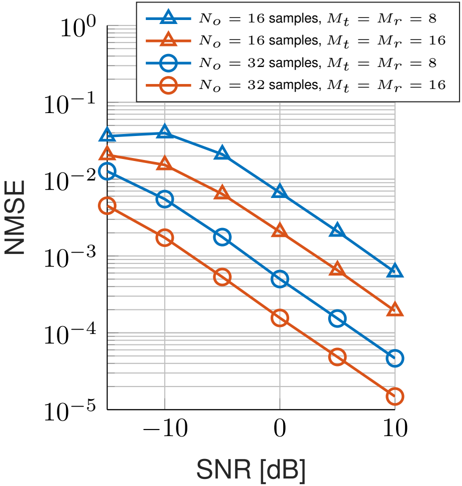

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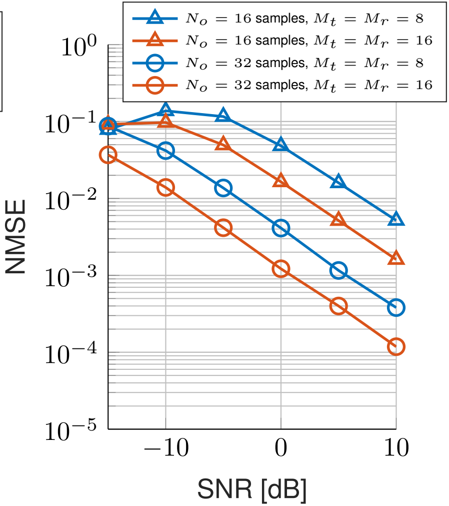

CFO

Conclusion

Part I

- Low-complexity tensor beamforming filters
- Low-rank extension of tensor filters

Part II

- Energy efficiency analysis of precoding structures for mmWave massive MIMO
- Double-sided massive MIMO transceiver schemes

Part III

- Tensor methods for joint wideband channel parameters, phase noise and CFO

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Some Perspectives

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  - Tensor train model
  - Unsupervised strategies
- Wide-band and multi-carrier extensions of the proposed massive MIMO methods
- Transceiver performance under imperfect CSI
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Publications

Journal Papers

1. IEEE JSTSP 2018 – *Energy efficiency of mmWave massive MIMO precoding with low-resolution DACs*
2. Signal Processing 2019 – *Separable linearly constrained minimum variance beamformers*
3. IET Signal Processing 2019 – *Low-complexity separable beamformers for massive antenna array systems*
4. Under revision – *Double-sided massive MIMO transceivers for mmWave communications*

Publications

Conference Papers

1. EUSIPCO'17 – *A low-complexity equalizer for massive MIMO systems based on array separability*
2. SBRT'18 – *Separable least-mean squares beamforming*
3. ICASSP'19 – *Tensor-based estimation of mmWave MIMO channels with carrier frequency offset*
4. ISWCS'19 – *Low-rank tensor MMSE equalization*
5. Under preparation – *Joint phase noise and carrier frequency offset estimation in wideband mmWave MIMO channels*

Doctoral Thesis Defense

Signal Processing Methods for Large-Scale Multi-Antenna Systems

Lucas Nogueira Ribeiro

Advisor: Prof. Dr. André Lima Férrer de Almeida
Co-Advisor: Prof. Dr. João César Moura Mota

Universidade Federal do Ceará
Teleinformatics Engineering Department

Fortaleza, October 10th, 2019