Doctoral Thesis Defense

Signal Processing Methods for Large-Scale Multi-Antenna Systems

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Motivation and Scope

• Number of connected devices keeps growing every year

• Very large capacity requirements

• How to achieve larger system capacity?
  • Beamforming gain → Massive MIMO
  • Increase bandwidth → Millimeter wave bands
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Challenges

1. Computational complexity of large-scale filter design;
2. Energy efficiency of mmWave massive MIMO transceivers;
3. MmWave channel estimation under synchronization impairments.
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Iterative implementation of linear filters

  - Systolic array implementation of QR decomposition for Zero-Forcing filtering

  - Joint steepest descent and Jacobi method detection
State of the Art – Problem 1

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Hybrid analog/digital (A/D) systems


Digital systems with low-resolution data converters

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State of the Art – Problem 3

MmWave channel estimation with carrier frequency offset (CFO) impairment

  - Sparse bilinear optimization $\rightarrow$ message passing solution

- J. Rodríguez-Fernández and N. González-Prelcic, “Channel estimation for hybrid mmWave MIMO systems with CFO uncertainties,” to appear in IEEE Transactions on Wireless Communications, 2019
  - Maximum likelihood estimator
### MmWave channel estimation with carrier frequency offset (CFO) impairment

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  - Maximum likelihood estimator
Part I: Multilinear Filtering

Related publications

• *IET Signal Processing*, v. 13, n. 4, p. 434–442, June 2019
  
  • *Signal Processing*, v. 158, p. 15–25, May 2019
    
    • *Proc. SBRT 2018*
      
      • *Proc. IEEE ISWCS 2019*
Multilinear Filtering

- Multi-linear and time-invariant filter:

\[ w = w_1 \otimes \cdots \otimes w_M \in \mathbb{C}^N \]

where \( w_m \in \mathbb{C}^{N_m} \) with \( \prod_{m=1}^{M} N_m = N \)

- Basic idea: design each factor instead of the whole vector
- Fewer computations?
- How much performance loss, if any?
- Beamforming and equalization problems
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Scenario

- Narrowband far-field propagation
- \( R \) independent sources \( s_r[k] \) impinging on the receiver with \( N \) antennas
- Multi-user system with \( R \) users and line-of-sight propagation
Multilinear Filtering

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### System Model

#### Received Signal

$$x[k] = As[k] + b[k]$$  \hspace{1cm} (1)

- \(s[k] = [s_1[k], \ldots, s_R[k]]^T \in \mathbb{C}^R\) – sources vector
- \(A = [a(\phi_1, \theta_1), \ldots, a(\phi_R, \theta_R)] \in \mathbb{C}^{N \times R}\) – array manifold matrix
- \(b[k] = [b_1[k], \ldots, b_N[k]]^T \in \mathbb{C}^N\) – ad. white Gaus. noise (AWGN)

#### Beamforming Filter

- Filter \(x[k]\) to recover a signal of interest (\(r = 1\))
- \(w = [w_1, \ldots, w_N]^T \in \mathbb{C}^N\)
- Filter output:
  $$y[k] = w^H x[k]$$
**System Model**

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Uniform Planar Array

UPA array response is **separable**

\[
\mathbf{a}(\phi_r, \theta_r) = \begin{bmatrix}
1 \\
e^{-j\pi \cos \theta_r} \\
\vdots \\
e^{-j\pi (N_v - 1) \cos \theta_r}
\end{bmatrix} \otimes \begin{bmatrix}
1 \\
e^{-j\pi \sin \phi_r \sin \theta_r} \\
\vdots \\
e^{-j\pi (N_h - 1) \sin \phi_r \sin \theta_r}
\end{bmatrix}
= \mathbf{a}_v(q_r) \otimes \mathbf{a}_h(p_r)
\]

where \( p_r = \sin \phi_r \sin \theta_r \) and \( q_r = \cos \theta_r \).

Array manifold matrix:

\[
\mathbf{A} = \mathbf{A}_v \otimes \mathbf{A}_h \in \mathbb{C}^{N_v N_h \times R}
\]

Apply separable filter \( \mathbf{w} = \mathbf{w}_v \otimes \mathbf{w}_h \) to each array dimension

\[
(\mathbf{w}_v \otimes \mathbf{w}_h)^\mathsf{H} (\mathbf{A}_v \otimes \mathbf{A}_h) = (\mathbf{w}_v^\mathsf{H} \mathbf{A}_v) \otimes (\mathbf{w}_h^\mathsf{H} \mathbf{A}_h)
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(w_v \otimes w_h)^H (A_v \otimes A_h) = (w_v^H A_v) \otimes (w_h^H A_h)
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(w_v \otimes w_h)^H (A_v \otimes A_h) = (w_v^H A_v) \otimes (w_h^H A_h)
\]
System Model – Tensor Formulation

• We define the array steering tensor

\[ \mathbf{A} = I_{3,R} \times_1 \mathbf{A}_h \times_2 \mathbf{A}_v \times_3 I_R \in \mathbb{C}^{N_h \times N_v \times R} \]  

(2)

• Received signal model

\[ \mathbf{X}[k] = \mathbf{A} \times_3 \mathbf{s}^T[k] + \mathbf{B}[k] \in \mathbb{C}^{N_h \times N_v} \]  

(3)

• Filter \( \mathbf{w} = \mathbf{w} = \mathbf{w}_v \otimes \mathbf{w}_h \) output:

\[ \mathbf{y}[k] = \mathbf{w}^H \mathbf{x}[k] = \mathbf{X}[k] \times_1 \mathbf{w}_h^H \times_2 \mathbf{w}_v^H \]

\[ = \mathbf{w}_h^H \mathbf{X}[k] \mathbf{w}_v^* = \mathbf{w}_v^H \mathbf{X}^T[k] \mathbf{w}_h^* \]  

(4)

(5)

• Define \( \mathbf{u}_h[k] = \mathbf{X}[k] \mathbf{w}_v^* \in \mathbb{C}^{N_h} \) and \( \mathbf{u}_v[k] = \mathbf{X}^T[k] \mathbf{w}_h^* \in \mathbb{C}^{N_v} \)

• Output signal rewritten as

\[ \mathbf{y}[k] = \mathbf{w}_h^H \mathbf{u}_h[k] = \mathbf{w}_v^H \mathbf{u}_v[k] \]  

(6)

• Output bilinear w.r.t. sub-filters
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Beamforming Filter Design – Tensor MMSE (TMMSE)

- Consider the classical minimum mean square error (MMSE) filter design:

\[
\min_w \mathbb{E} \left[ \left| s_{SOI}[k] - w^H x[k] \right|^2 \right] \tag{7}
\]

- From the bilinearity property, we may write

\[
\min_{w_h} \mathbb{E} \left[ \left| s_{SOI}[k] - w_h^H u_h[k] \right|^2 \right] \tag{8a}
\]

\[
\min_{w_v} \mathbb{E} \left[ \left| s_{SOI}[k] - w_v^H u_v[k] \right|^2 \right] \tag{8b}
\]

- Alternating optimization in (8a) and (8b) until convergence

- After convergence\(^\dagger\): \(w_{TMMSE} = w_v \otimes w_h\)

- Tikhonov regularization is applied to avoid numerical instability

- Exchange degrees of freedom for complexity reduction

\*\(\text{N (linear) vs. min(N}_h, N_v) \text{ (tensor)}\)

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- Consider the classical minimum mean square error (MMSE) filter design:

  $$\min_w \mathbb{E} \left[ |s_{SOI}[k] - w^H x[k]|^2 \right]$$  \hspace{1cm} (7)

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\min_w \mathbb{E} \left[ \left| s_{\text{SOI}}[k] - w^H x[k] \right|^2 \right]
\]  

(7)

- From the bilinearity property, we may write

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\min_{w_h} \mathbb{E} \left[ \left| s_{\text{SOI}}[k] - w_h^H u_h[k] \right|^2 \right]
\]  

(8a)

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\min_{w_v} \mathbb{E} \left[ \left| s_{\text{SOI}}[k] - w_v^H u_v[k] \right|^2 \right]
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• Exchange **degrees of freedom** for complexity reduction

\[ N \text{ (linear)} \text{ vs. } \min(N_h, N_v) \text{ (tensor)} \]

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\] (8a)

\[
\min_{w_v} E \left[ \left| s_{SOI}[k] - w_v^H u_v[k] \right|^2 \right]
\] (8b)

- Alternating optimization in (8a) and (8b) until convergence

- After convergence\(^1\): \(w_{TMMSE} = w_v \otimes w_h\)

- Tikhonov regularization is applied to avoid numerical instability

- Exchange **degrees of freedom** for complexity reduction

- \(N\) (linear) vs. \(\min(N_h, N_v)\) (tensor)

---

Beamforming Filter Design – Tensor LCMV (TLCMV)

• We also consider the linear constraint minimum variance (LCMV) filter

\[
\min_w w^H R_{xx} w, \quad \text{s.t. } C^H w = f
\]

(9)

where \( C \in \mathbb{C}^{N \times R} \) denotes the constraint matrix, \( f \in \mathbb{C}^R \) the array factor vector and \( R_{xx} \) the cov. matrix of \( x[k] \)

• We can decouple (9) into

\[
\min_{w_h} w_h^H R_{hh} w_h, \quad \text{s.t. } C_h^H w_h = f_h \quad (10a)
\]

\[
\min_{w_v} w_v^H R_{vv} w_v, \quad \text{s.t. } C_v^H w_v = f_v \quad (10b)
\]

• Apply alternating optimization to (10) until convergence
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\min_{w_v} w_v^H R_{vv} w_v, \quad \text{s.t. } C_v^H w_v = f_v \]  

(10b)

• Apply alternating optimization to (10) until convergence
System Model – Sub-array Formulation

- Linear sub-arrays in planar array
  
  \[ x_h[k] = A_h s[k] + b_h[k] \in \mathbb{C}^{N_h} \quad (11) \]

- Horizontal sub-array

- Vertical sub-array
  
  \[ x_v[k] = A_v s[k] + b_v[k] \in \mathbb{C}^{N_v} \quad (12) \]

- Idea: design \( w_h \) and \( w_v \) independently

- Capture sub-array signals only

- Obtain full beamformer by
  
  \[ w = w_v \otimes w_h \]
System Model – Sub-array Formulation

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- **Idea:** design \( w_h \) and \( w_v \) independently
- Capture sub-array signals only
- Obtain full beamformer by
  \[ w = w_v \otimes w_h \]
Kronecker MMSE (KMMSE) Filter

\[
\min_{\mathbf{w}_h} \mathbb{E} \left[ |s_{\text{SOI}}[k] - \mathbf{w}_h^H \mathbf{x}_h[k]|^2 \right] \quad (13a)
\]

\[
\min_{\mathbf{w}_v} \mathbb{E} \left[ |s_{\text{SOI}}[k] - \mathbf{w}_v^H \mathbf{x}_v[k]|^2 \right] \quad (13b)
\]

Kronecker LCMV (KLCMV) Filter

\[
\min_{\mathbf{w}_h} \mathbf{w}_h^H \mathbf{R}_h \mathbf{w}_h, \quad \text{s.t.} \quad \mathbf{C}_h^H \mathbf{w}_h = \mathbf{f}_h \quad (14a)
\]

\[
\min_{\mathbf{w}_v} \mathbf{w}_v^H \mathbf{R}_v \mathbf{w}_v, \quad \text{s.t.} \quad \mathbf{C}_v^H \mathbf{w}_v = \mathbf{f}_v \quad (14b)
\]

where \( \mathbf{R}_h \) and \( \mathbf{R}_v \) are the covariance matrices of \( \mathbf{x}_h[k] \) and \( \mathbf{x}_v[k] \), respectively.

Compute \( \mathbf{w}_h, \mathbf{w}_v \) and combine with Kronecker once!
## Beamforming Filter Design – Kronecker Filters

### Kronecker MMSE (KMMSE) Filter

\[
\min_{w_h} \mathbb{E} \left[ | s_{SOI}[k] - w_h^H x_h[k] |^2 \right] \quad (13a)
\]

\[
\min_{w_v} \mathbb{E} \left[ | s_{SOI}[k] - w_v^H x_v[k] |^2 \right] \quad (13b)
\]

### Kronecker LCMV (KLCMV) Filter

\[
\min_{w_h} w_h^H R_h w_h, \quad \text{s.t.} \quad C_h^H w_h = f_h \quad (14a)
\]

\[
\min_{w_v} w_v^H R_v w_v, \quad \text{s.t.} \quad C_v^H w_v = f_v \quad (14b)
\]

where \( R_h \) and \( R_v \) are the covariance matrices of \( x_h[k] \) and \( x_v[k] \), respectively.

Compute \( w_h, w_v \) and combine with Kronecker once!
Beamforming Filter Design – Kronecker Filters

Kronecker MMSE (KMMSE) Filter

\[
\begin{align*}
\min_{w_h} & \mathbb{E} \left[ \left| s_{\text{SOI}}[k] - w_h^H x_h[k] \right|^2 \right] \\
\min_{w_v} & \mathbb{E} \left[ \left| s_{\text{SOI}}[k] - w_v^H x_v[k] \right|^2 \right]
\end{align*}
\] (13a) (13b)

Kronecker LCMV (KLCMV) Filter

\[
\begin{align*}
\min_{w_h} & \ w_h^H R_h w_h, \quad \text{s.t.} \quad C_h^H w_h = f_h \\
\min_{w_v} & \ w_v^H R_v w_v, \quad \text{s.t.} \quad C_v^H w_v = f_v
\end{align*}
\] (14a) (14b)

where \( R_h \) and \( R_v \) are the covariance matrices of \( x_h[k] \) and \( x_v[k] \), respectively.

Compute \( w_h, w_v \) and combine with Kronecker once!
Beamforming Filter Design

Computational Complexity

- **MMSE/LCMV**: $O(N^3)$
- **TMMSE/TLCMV**: $O(I(N_h^3 + N_v^3))$, for $I$ iterations
- **KMMSE/KLCMV**: $O(N_h^3 + N_v^3)$

- The MMSE and LCMV filters (as well as their tensor extensions) depend on second-order statistics

- **Sample estimates when they are not known**

- The adaptive implementation of the proposed tensor and Kronecker MMSE and LCMV filters have been developed
Beamforming Filter Design

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Simulation Results

Setup

- Direction cosines $p_r$ and $q_r$ uniformly distributed in $\mathcal{U}(-0.9, 0.9)$
- $R = 4$ sources QPSK signals
- $N = 64$ antennas ($N_h = N_v = 8$), half-wave spacing

Figures of Merit

- Floating point operations (flops) – computational complexity
- Uncoded bit error ratio (BER) for MMSE-type filters
- Output SINR for LCMV-type filters

$$\text{SINR}_{\text{out}} = \frac{w^H R_{dd} w}{w^H (R_{ii} + R_{bb}) w}.$$
## Simulation Results

### Setup
- Direction cosines $p_r$ and $q_r$ uniformly distributed in $\mathcal{U}(-0.9, 0.9)$
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\[
\text{SINR}_{\text{out}} = \frac{w^H R_{dd} w}{w^H (R_{ii} + R_{bb}) w}.
\]
Simulation Results – Computational Complexity

- MMSE
- TMMSE
- KMMSE

Number of flops

- LCMV
- TLCMV
- KLCMV

$N_h \times N_v$
Simulation Results – Computational Complexity

Number of flops vs. $N_h \times N_v$

- MMSE
- TMMSE
- KMMSE

Number of flops vs. $N_h \times N_v$

- LCMV
- TLCMV
- KLCMV
Simulation Results – Computational Complexity

- **MMSE**
- **TMMSE**
- **KMMSE**

Number of flops vs. $N_h \times N_v$

- **LCMV**
- **TLCMV**
- **KLCMV**
Simulation Results – Computational Complexity

![Graph showing number of flops vs. Nh \times Nv for MMSE, TMMSE, and KMMSE.](image)

![Graph showing number of flops vs. Nh \times Nv for LCMV, TLCMV, and KLCMV.](image)
Simulation Results – Computational Complexity

![Graph showing computational complexity](image)

- Number of flops vs. $N_h \times N_v$
- Graphs for MMSE, TMMSE, KMMSE, LCMV, TLCMV, KLCMV

**Figure**: Simulation results illustrating computational complexity for different algorithms as a function of $N_h \times N_v$. The graphs compare MMSE, TMMSE, and KMMSE on the left, and LCMV, TLCMV, and KLCMV on the right, showing how computational requirements scale with respect to $N_h$ and $N_v$.
Simulation Results – Computational Complexity

Number of flops

\[ N_h \times N_v \]

\[ 16 \quad 64 \quad 144 \quad 256 \]

\[ 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8 \]

- MMSE
- TMMSE
- KMMSE

\[ N_h \times N_v \]

\[ 0 \quad 100 \quad 200 \quad 300 \]

\[ 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8 \]

- LCMV
- TLCMV
- KLCMV
Simulation Results – BER and SINR

- BER
  - MMSE
  - TMMSE
  - KMMSE ($\delta = 0.5$)

- Output SINR [dB]
  - LCMV
  - TLCMV
  - KLCMV

SNR [dB]

BER

SNR [dB]
Simulation Results – BER and SINR

BER

\[-20\, \text{dB} \quad -10\, \text{dB} \quad 0 \quad 10 \quad 20\]

SNR [dB]

- $10^0$
- $10^{-1}$
- $10^{-2}$
- $10^{-3}$
- $10^{-4}$
- $10^{-5}$
- $10^{-6}$

KMMSE $(\delta = 0.5)$

MMSE

TMMSE

Output SINR [dB]

\[-10 \quad 0 \quad 10 \quad 20 \quad 30 \quad 40\]

SNR [dB]

LCMV

TLCMV

KLCMV
Simulation Results – BER and SINR

![Graph showing BER and SINR vs SNR for different types of equalization: MMSE, TMMSE, KMMSE, LCMV, TLCMV, KLCMV. The graph displays the BER on a logarithmic scale (y-axis) against the SNR in dB (x-axis) for various SINR values, with different markers and colors for each equalization method.](image-url)
Simulation Results – BER and SINR

![Graph showing BER and SINR results for different methods (MMSE, TMMSE, KMMSE, LCMV, TLCMV, KLCMV) across various SNR values. The plots demonstrate the performance of these methods with decreasing BER and increasing SINR as SNR increases.]
Simulation Results – BER and SINR

![Graph showing BER vs. SNR for different algorithms: MMSE, TMMSE, KMMSE (δ = 0.5), LCMV, TLCMV, KLCMV. The graphs depict the performance of these algorithms with decreasing BER as SNR increases.](image-url)
Simulation Results – BER and SINR

- BER
  - MMSE
  - TMMSE
  - KMMSE ($\delta = 0.5$)

- SNR [dB]
- Output SINR [dB]

- LCMV
- TLCMV
- KLCMV
What about non-separable channels? Multipath?

Low-Rank Filters

\[ w = \sum_{r=1}^{R} w_{1,r} \otimes \ldots \otimes w_{M,r} \]

Order \( M \)

Rank \( R \)
What about non-separable channels? Multipath?

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Low-Rank Filters

\[ w = \sum_{r=1}^{R} w_{1,r} \otimes \cdots \otimes w_{M,r} \]

Order \( M \)
Rank \( R \)
System Model

- Uplink scenario, $U$ users

$$\mathbf{x}[k] = \sum_{u=1}^{U} \mathbf{H}_u \mathbf{s}_u[k] + \mathbf{b}[k]$$  \hspace{1cm} (15)

$$\mathbf{s}_u[k] = [s_u[k], \ldots, s_u[k-Q+1]]^T$$  \hspace{1cm} (16)

- Channel model

$$\mathbf{H}_u = \sum_{\ell=1}^{L} \alpha_{u,\ell} \mathbf{a}(\theta_{u,\ell}) \mathbf{g}(\tau_{u,\ell})^T \in \mathbb{C}^{N \times Q}$$  \hspace{1cm} (17)

$$\mathbf{a}(\theta_{u,\ell}) = [1, \ldots, e^{-j\pi(N-1)} \cos \theta_{u,\ell}]^T \in \mathbb{C}^N$$  \hspace{1cm} (18)

$$\mathbf{g}(\tau_{u,\ell}) = [g(-\tau_{u,\ell}), \ldots, g((Q-1)T-\tau_{u,\ell})]^T \in \mathbb{C}^Q$$  \hspace{1cm} (19)

- $\mathbf{H}_u$ is not separable, but admits a low-rank structure

- Low-rank equalizer to filter the desired data stream $s_u[k]$
System Model

- Uplink scenario, \( U \) users

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\mathbf{x}[k] = \sum_{u=1}^{U} \mathbf{H}_u \mathbf{s}_u[k] + \mathbf{b}[k]
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- $\mathbf{H}_u$ is not separable, but admits a **low-rank** structure

- **Low-rank equalizer** to filter the desired data stream $s_u[k]$
System Model

• Uplink scenario, $U$ users

$$\mathbf{x}[k] = \sum_{u=1}^{U} \mathbf{H}_u \mathbf{s}_u[k] + \mathbf{b}[k]$$

(15)

$$\mathbf{s}_u[k] = [s_u[k], \ldots, s_u[k - Q + 1]]^T$$

(16)

• Channel model

$$\mathbf{H}_u = \sum_{\ell=1}^{L} \alpha_{u,\ell} \mathbf{a}(\theta_{u,\ell}) \mathbf{g}(\tau_{u,\ell})^T \in \mathbb{C}^{N \times Q}$$

(17)

$$\mathbf{a}(\theta_{u,\ell}) = \left[1, \ldots, e^{-j\pi(N-1)\cos \theta_{u,\ell}}\right]^T \in \mathbb{C}^N$$

(18)

$$\mathbf{g}(\tau_{u,\ell}) = [g(-\tau_{u,\ell}), \ldots, g((Q - 1)T - \tau_{u,\ell})]^T \in \mathbb{C}^Q$$

(19)

• $\mathbf{H}_u$ is not separable, but admits a low-rank structure

• Low-rank equalizer to filter the desired data stream $s_u[k]$
Some Algebra...

The filter coefficients can be written as

$$w_{n_1,\ldots,n_D} = \sum_{r=1}^{R} \prod_{d=1}^{D} [w_{d,r}]_{n_d}, \quad (20)$$

which allows us to recast the equalizer output $y[k] = w^H x[k]$ as follows

$$y[k] = \sum_{n_1,\ldots,n_D=1}^{N_1,\ldots,N_D} \left( \sum_{r=1}^{R} [w_{1,r}]^{*}_{n_1} \cdots [w_{D,r}]^{*}_{n_D} \right) x_{n_1,\ldots,n_D}[k]. \quad (21a)$$

$$= \sum_{r=1}^{R} \sum_{n_d=1}^{N_d} [w_{d,r}]^{*}_{n_d} \left( \sum_{n_q=1}^{N_q} \prod_{q \neq d}^{D} [w_{q,r}]^{*}_{n_q} x_{n_1,\ldots,n_D}[k] \right) \quad (21b)$$

$$= \sum_{r=1}^{R} \sum_{n_d=1}^{N_d} [w_{d,r}]^{*}_{n_d} [u_{d,r}[k]_{n_d} = w^H_d u_d[k] \quad (21c)$$

Output is linear w.r.t. each tensor filter factor $w_d$!
Low-Rank Tensor MMSE

- We formulate for each filter mode

\[
\min_{\mathbf{w}_d} \mathbb{E} \left[ |s_u[k - \delta] - \mathbf{w}_d^H \mathbf{u}_d[k]|^2 \right], \quad d \in \{1, \ldots, D\}.
\]

where

\[
\mathbf{u}_d[k] = [\mathbf{u}_{d,1}^T[k], \ldots, \mathbf{u}_{d,R}^T[k]]^T \in \mathbb{C}^{RN_d} \tag{22}
\]

\[
\mathbf{u}_{d,r}[k] = \mathbf{X}_{(d)}[k] \bigotimes_{q \neq d} \mathbf{w}_{q,r}^* \in \mathbb{C}^{N_d} \tag{23}
\]

\[
\mathbf{w}_d = [\mathbf{w}_{d,1}^T, \ldots, \mathbf{w}_{d,R}^T]^T \in \mathbb{C}^{RN_d} \tag{24}
\]

- Solution:

\[
\mathbf{w}_{d,\text{MMSE}} = \mathbf{R}_{u_d,u_d}^{-1} \mathbf{p}_{u_d} \in \mathbb{C}^{RN_d}, \tag{25}
\]

\[
\mathbf{R}_{u_d,u_d} = \mathbb{E} [\mathbf{u}_d[k] \mathbf{u}_d^H[k]] \in \mathbb{C}^{RN_d \times RN_d}, \tag{26}
\]

\[
\mathbf{p}_{u_d} = \mathbb{E} [\mathbf{u}_d[k] s_u^*[k - \delta]] \in \mathbb{C}^{RN_d} \tag{27}
\]

- Alternating optimization
Computational Complexity

- $N$: number of antennas
- $K$: number of snapshots (covariance matrix estimation)
- MMSE filter
  \[
P_{\text{MMSE}}(N, K) = \frac{N^2K + NK}{\text{statistics estimation}} + O(N^3) + \frac{N^2}{\text{filtering}}
\]
- LR-TMMSE filter
  \[
P_{\text{LR-TMMSE}}(\{N_d\}, D, I, K) =
  I \left[ \sum_{d=1}^{D} \frac{R(D - 1)NK + N_d^2K + N_dK}{\text{statistics estimation}} + O(N_d^3) + \frac{N_d^2}{\text{filtering}} \right]
\]
- $I$: iterations number
- **Tensor overhead!**
Computational Complexity

- $N$: number of antennas
- $K$: number of snapshots (covariance matrix estimation)
- MMSE filter

\[
P_{\text{MMSE}}(N, K) = N^2K + NK + O(N^3) + N^2
\]

- LR-TMMSE filter

\[
P_{\text{LR-TMMSE}}(\{N_d\}, D, I, K) = I \left[ \sum_{d=1}^{D} \frac{R(D - 1)NK + N_d^2K}{\text{statistics estimation}} + O(N_d^3) + N_d^2 \right]
\]

- $I$: iterations number
- **Tensor overhead!**
Computational Complexity

- $N$: number of antennas
- $K$: number of snapshots (covariance matrix estimation)
- MMSE filter

$$P_{\text{MMSE}}(N, K) = \underbrace{N^2K + NK}_{\text{statistics estimation}} + \underbrace{O(N^3)}_{\text{cov. matrix inversion}} + \underbrace{N^2}_{\text{filtering}}$$

- LR-TMMSE filter

$$P_{\text{LR-TMMSE}}(\{N_d\}, D, I, K) = I \left[ \sum_{d=1}^{D} R(D - 1)NK + N_d^2K + N_dK + \underbrace{O(N_d^3)}_{\text{cov. matrix inversion}} + \underbrace{N_d^2}_{\text{filtering}} \right]$$

- $I$: iterations number

- Tensor overhead!
Computational Complexity

- \( N \): number of antennas
- \( K \): number of snapshots (covariance matrix estimation)
- MMSE filter

\[
P_{\text{MMSE}}(N, K) = N^2K + NK + O(N^3) + N^2
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- LR-TMMSE filter

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I \left[ \sum_{d=1}^{D} R(D - 1)NK + N_d^2K + N_dK + O(N_d^3) + N_d^2 \right]
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- \( I \): iterations number
- Tensor overhead!
Number of products

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LR-TMMSE (\(D = 2\))

LR-TMMSE (\(D = 3\))

LR-TMMSE (\(D = 4\))

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N = 512, I = 2, R = 3

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\text{SINR}(w) = \frac{w^H R_{xx} w}{w^H (R_{ii} + R_{bb}) w}
\]

\( N = 512 \) antennas

\( \text{SNR} = 20 \text{ dB} \)

Filter order \( D = 3 \)

\( U = 4 \) users

\( L = 4 \) paths
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(28)

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\(N = 512\) antennas

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\( N = 512 \) antennas

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Part II: MmWave Massive MIMO Transceiver Design

Related publications


• *IEEE Access (under revision)*
Massive MIMO Precoding – Energy Efficiency

(a) Fully-digital

(b) Hybrid analog/digital

Large-scale antenna arrays at transmitting side

- Challenges: power consumption, energy efficiency
- Fully digital vs. hybrid A/D (fully- and partially-connected)
- Low-res. data converters → pow. amps. close to saturation (more efficient)

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- Single-user mmWave MIMO system with $N_r \times N_t$ antennas

- Received signal:
  $$y = Hx + n \in \mathbb{C}^{N_r}$$  \hfill (29)

- Transmitted signal with DAC and RF losses:
  $$x = \frac{1}{\sqrt{L_{RF}}} F_{RF} Q_b( F_{BB} s ) = \frac{1}{\sqrt{L_{RF}}} \tilde{x} \in \mathbb{C}^{N_t} ,$$  \hfill (30)

  $L_{RF}$ RF losses, $L_t$ TX RF chains, $F_{RF} \in \mathbb{C}^{N_t \times L_t}$ and $F_{BB} \in \mathbb{C}^{L_t \times N_s}$ analog and baseband precoders, resp.

- Channel model
  $$H = \sqrt{\frac{N_t N_r}{L}} \sum_{\ell=1}^{L} \alpha_{\ell} a_r \left( \phi_{\ell}^{(r)}, \theta_{\ell}^{(r)} \right) a_t \left( \phi_{\ell}^{(t)}, \theta_{\ell}^{(t)} \right)^{\mathrm{H}} \in \mathbb{C}^{N_r \times N_t} ,$$  \hfill (31)
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- Proposed quantized signal model

\[ y \approx \frac{1}{\sqrt{L_{RF}}} H' u + n_G = \frac{1}{\sqrt{L_{RF}}} H_{eq} \Upsilon_b F_{BB} s + n_G. \]  

(32)

where

- \( H' = H_{eq} \Upsilon_b \in \mathbb{C}^{N_r \times M} \) stands for channel + DAC distortion matrix
- \( H_{eq} = HF_{RF} \in \mathbb{C}^{N_r \times M} \) denotes the equivalent channel
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- \( u = F_{BB} s \in \mathbb{C}^M \) baseband-precoded signal
- \( n_G \) additive noise

- Covariance matrix of \( n_G \) is given by

\[ R_{n_G n_G} = \frac{1}{L_{RF}} H_{eq} R_{ee} H_{eq}^H + R_{nn} \in \mathbb{C}^{N_r \times N_r} \]  

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\[ R_{ee} = \rho_b \text{diag}(R_{uu}) \in \mathbb{C}^{M \times M} \]  

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- Noise covariance matrix depends on the input signal (causality problem?)
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Quantized Precoding Problem

Problem Formulation

Assuming perfect channel state information (CSI) and whitening:

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\begin{align*}
\text{maximize} & \quad \log_2 \det \left( I_{N_r} + \frac{1-\rho_b}{L_{RF}} R_{n_G n_G}^{-1/2} H F_{RF} F_{BB} F_{BB}^H H^H R_{n_G n_G}^{-1/2, H} \right) \\
\text{subject to} & \quad [F_{RF}]_{u,v} \in F_{RF}, \forall u \forall v, \mathbb{E} \left[ \|\tilde{x}\|_2^2 \right] \leq P_{\text{max}}.
\end{align*}
\]

(35)

where \( \tilde{x} = F_{RF} \Upsilon_b u + F_{RF} e \in \mathbb{C}^{N_t} \)

- It is general to model the (un)quantized hybrid and fully-digital precoding problems
- Sub-optimal solution: optimize \( F_{RF} \) and \( F_{BB} \) independently

Analog Precoder \( F_{RF} \) Design

- Fully-connected: alternating projection method\(^2\)
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Quantized Precoding Problem

Problem Formulation

Assuming perfect channel state information (CSI) and whitening:

\[
\begin{align*}
&\text{maximize } F_{RF}, F_{BB} \quad \log_2 \det \left( I_{N_r} + \frac{1-\rho_b}{L_{RF}} R_{n_G}^{-1/2} H F_{RF} F_{BB} F_{BB}^H F_{RF} H^H R_{n_G}^{-1/2,H} \right) \\
&\text{subject to } [F_{RF}]_{u,v} \in \mathcal{F}_{RF}, \forall u \forall v, \mathbb{E} \left[ \|\tilde{x}\|_2^2 \right] \leq P_{\text{max}}.
\end{align*}
\]

where \( \tilde{x} = F_{RF} \Upsilon_b u + F_{RF} e \in \mathbb{C}^{N_t} \)

- It is general to model the (un)quantized hybrid and fully-digital precoding problems
- Sub-optimal solution: optimize \( F_{RF} \) and \( F_{BB} \) independently

Analog Precoder \( F_{RF} \) Design

- Fully-connected: alternating projection method\(^2\)
- Partially-connected: maximum eigenmode transmission by power method

---

Quantized Precoding Problem

Baseband Precoder $F_{BB}$ Design

- Design baseband filter as optimal precoder in infinite-resolution DAC scenarios

\[
\begin{align*}
\text{maximize} & \quad \log_2 \det \left( I_{N_r} + R_{nn}^{-1} H_{eq} F_{BB} F_{BB}^H H_{eq}^H \right) \\
\text{subject to} & \quad \mathbb{E} [\|\tilde{x}\|_2^2] \leq P_{\text{max}}.
\end{align*}
\] (36)

- Avoids causality problem in total noise covariance matrix
- Consider the SVD of the equivalent channel: $H_{eq} = U \Sigma V^H$
- SVD precoding + waterfilling power allocation:

\[
F_{BB} = \sqrt{P_{\text{max}}} \frac{Q}{\|F_{RF}Q\|_F} Q
\] (37)

\[
Q = V \Lambda^{1/2} \in \mathbb{C}^{M \times N_s}
\] (38)

where $\Lambda \in \mathbb{R}^{N_s \times N_s}$ denotes the diagonal power allocation matrix.
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Power Consumption and Loss Models

Power Consumption Formulas

- Fully-digital: \( P_D = P_{LO} + P_{PA} + N_t[2P_{DAC}(b_{DAC}, F_s) + P_{RF}] \)

- Hybrid A/D fully-connected:
  \( P_{FPSN} = P_{LO} + P_{PA} + L_t[2P_{DAC}(b_{DAC}, F_s) + P_{RF}] + N_t L_t P_{PS}(b_{PS}) \)

- Hybrid A/D partially-connected:
  \( P_{PPSN} = P_{LO} + P_{PA} + L_t[2P_{DAC}(b_{DAC}, F_s) + P_{RF}] + N_a L_t P_{PS}(b_{PS}) \)

- Power amplifier: \( P_{PA} = P_x/\eta \), for efficiency \( \eta \) and
  \[
P_x = \frac{1}{L_{RF}} [(1 - \rho_b)\|F_{RF}F_{BB}\|^2_F + \text{Tr}(F_{RF}R_{ee}F_{RF}^{H})]
  \]

RF Devices Loss

- 2-way pow. div: \( L_D(N_t) \)
- 2-way pow. comb: \( L_C(L_t) \)
- Phase-shifter (passive or active): \( L_{PS} \)

Phase-Shifting Network Loss

- \( L_{RF}^{FPSN} = L_D(N_t)L_{PS}L_C(L_t) \)
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### Power Consumption and Loss Models

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Simulation Setup

- $N_t = 64$ and $N_r = 4$ antennas
- $L_t = 4$ RF chains
- $N_s = 4$ data streams
- $L = 5$ channel paths
- $P_{\text{max}} = 1$ W
- Phase shifter resolution: 5 bits
- DAC sampling frequency $F_s = 1$ GHz
- Energy efficiency: 
  \[ \frac{\text{spectral efficiency}}{\text{power consumption}} \quad \text{[bit/J]} \]

Phase shifter implementation

- Active: \( \uparrow \) power consumption  
  \( \downarrow \) insertion loss
- Passive: \( \downarrow \) power consumption  
  \( \uparrow \) insertion loss

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{PA}}$</td>
<td>$P_x/\eta$, $\eta = 27%$</td>
</tr>
<tr>
<td>$P_{\text{PS}}$</td>
<td>21.6 ; 0 mW</td>
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<tr>
<td>$P_{\text{LO}}$</td>
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</tr>
<tr>
<td>$P_{\text{H}}$</td>
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<tr>
<td>$P_{\text{M}}$</td>
<td>0.3 mW</td>
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<tr>
<td>$P_{\text{LP}}$</td>
<td>14 mW</td>
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<tr>
<td>$P_{\text{RF}}$</td>
<td>31.6 mW</td>
</tr>
<tr>
<td>$L_D$</td>
<td>0.6 dB</td>
</tr>
<tr>
<td>$\bar{L}_C$</td>
<td>0.6 dB + 3 dB</td>
</tr>
<tr>
<td>$L_{\text{PS}}$</td>
<td>$-2.3$ ; 8.8 dB</td>
</tr>
</tbody>
</table>
Simulation Results

Considering RF hardware losses
Simulation Results

Average spectral efficiency [bit/s/Hz]

SNR [dB]

Considering RF hardware losses

- $b_{DAC} = 8$
- $b_{DAC} = 1$
Simulation Results

Considering RF hardware losses
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Considering RF hardware losses

Average spectral efficiency [bit/s/Hz]

SNR [dB]

$b_{DAC} = 8$

$b_{DAC} = 1$
Simulation Results

![Graph showing SNR vs. Average spectral efficiency for different DAC resolutions and filter types.]

- SNR [dB]
- Average spectral efficiency [bit/s/Hz]
- Optimal unconstrained
- HPF (active)
- HPF (passive)
- HPP (active)
- HPP (passive)

Considering RF hardware losses

$\nu_{DAC} = 8$

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Consideration of RF hardware losses
Simulation Results

Considering RF hardware losses
Simulation Results

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SNR = 0 dB, $b_{\text{DAC}} \in \{1, \ldots, 8\}$
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Massive MIMO only at transmitter (base station)

Double-Sided Massive MIMO

Transceiver Design
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Transceiver Design
Double-Sided Massive MIMO

• Why?
  • Potentially better performance than canonical massive MIMO
  • Wireless backhauling, terahertz communications, among others

• Contributions
  • Low-complexity transceiver schemes with practical CSI requirements
  • Performance evaluation under different propagation conditions
Double-Sided Massive MIMO

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System Model

Signal and Channel Models

Downlink operation, 1 BS ($N_t$ antennas), $U$ UEs ($N_r$ antennas, each)

$$y_u = W_u^H H_u F_u s_u + \sum_{j=1 \atop j \neq u}^{U} W_u^H H_u F_j s_j + W_u^H b_u \in \mathbb{C}^{N_s},$$  \hspace{1cm} \text{(39)}

$$H_u = \sqrt{N_t N_r \over L} \sum_{\ell=1}^{L} \alpha_{\ell,u} \begin{pmatrix} a_{r,u} \left( \phi_{\ell}^{(r,u)}, \theta_{\ell}^{(r,u)} \right) \end{pmatrix} \begin{pmatrix} a_{t,u}^T \left( \phi_{\ell}^{(t,u)}, \theta_{\ell}^{(t,u)} \right) \end{pmatrix}$$  \hspace{1cm} \text{(40)}

Multi-Layer Filtering

Two layers: outer and inner layers

- $F_u = \gamma_u F_{o,u} F_{i,u}$, $F_{o,u} \in \mathbb{C}^{N_t \times M_t}$ and $\gamma_u F_{i,u} \in \mathbb{C}^{M_t \times N_s}$
- $W_u = W_{o,u} W_{i,u}$, $W_{o,u} \in \mathbb{C}^{N_r \times M_r}$ and $W_{i,u} \in \mathbb{C}^{M_r \times N_s}$

Each layer, one objective:

- Outer layer: increase SNR
- Inner layer: cancel multi-user interference
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Each layer, one objective:

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Signal Model (inner filters and effective channels)

Form low-dimensional effective channels!

\[
H_{\text{eff},u,j} = W_{o,u}^{H} H_{u} F_{o,j} \in \mathbb{C}^{M_{r} \times M_{t}}, \quad \gamma_{u} = \frac{\sqrt{P_{t}/U}}{\| F_{o,u} F_{i,u} \|_{F}} \tag{41}
\]

\[
y_{u} = \gamma_{u} W_{i,u}^{H} H_{\text{eff},u} F_{i,u} s_{u} + \sum_{j=1}^{U} \gamma_{j} W_{i,u}^{H} H_{\text{eff},u,j} F_{i,j} s_{j} + W_{i,u} b_{\text{eff},u} \tag{42}
\]
CSI Acquisition

First Stage: Outer Layer

CSI necessary for outer layer design

- Statistical CSI (uplink and downlink cov. matrices); or
- Partial CSI: path power and angles

**Depend only on macroscopic channel parameters!**

Second Stage: Inner Layer

- Estimate low-dimensional effective channels $H_{\text{eff},u,j}$
- Example: classical MMSE estimators

Time Scales

- Macroscopic: update outer layers
- Microscopic: update inner layers (low complexity!)
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• Obtain outer layer filters – **increase SNR**
  • Covariance matrix eigenfilter (CME)
  • Power-dominant path selection (PPS)
  • Semi-orthogonal path selection (SPS)

• Form inner layer filters – **cancel multi-user interference out**
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Simulation Results – Setup

- Investigate multiplexing capabilities
- Achievable sum rate as figure of merit
- Channel conditions
  - Poor: $L = 8$ rays
  - Rich: $L = 64$ rays
- Outer layer simulations: effect of number of streams $N_s$ on sum rate with single-user $U = 1$
- Inner layer simulations: influence of number $U$ of UEs on sum rate ($N_s = 1$)
- Benchmark: single-layer equivalent, partial zero-forcing $^3$
- Some parameters:
  - $N_t = N_r = 64$ antennas
  - Channel gains variance $\sigma^2_\alpha = 1$
  - 1000 independent experiments

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Simulation results – Outer layer

Achievable sum rate [bit/s/Hz]

Number of streams scaling $N_s/L$

Poor scattering ($L = 8$ paths)

Rich scattering ($L = 64$ paths)
Simulation results – Outer layer

Achievable sum rate [bit/s/Hz] vs Number of streams scaling $N_s/L$

Poor scattering ($L = 8$ paths)

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Achievable sum rate [bit/s/Hz] vs. Number of streams scaling $N_s/L$

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Simulation results – Outer layer

![Graph showing achievable sum rate vs. number of streams scaling $N_s/L$.](image)

**Poor scattering ($L = 8$ paths):**

- **CME**
- **SPS**
- **PPS**

**Rich scattering ($L = 64$ paths):**

- **CME**
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- **PPS**
Simulation results – Outer layer

Achievable sum rate [bit/s/Hz] vs. Number of streams scaling $N_s/L$

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Simulation Results

\[ U = 4 \text{ users}, \ M_t = M_r = 4 \]

\[ U = 32 \text{ users}, \ M_t = M_r = 4 \]
Simulation Results

$U = 4$ users, $M_t = M_r = 4$

$U = 32$ users, $M_t = M_r = 4$
Simulation Results

Achievable sum rate [bit/s/Hz] vs. SNR [dB]

- **MET-MMSE**
- **MET-BD**
- **BD-MER**
- **MET-MER**

For $U = 4$ users, $M_t = M_r = 4$

For $U = 32$ users, $M_t = M_r = 4$

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Part III: MmWave Channel Estimation with Synchronization Impairments

Related publications

• Proc. IEEE ICASSP 2019

• Wideband extension under preparation
MmWave Channel Estimation with Synchronization Impairments

• High-quality oscillators in mmWave are expensive

• Carrier signal far from ideal

• Impairments:
  • Carrier frequency offset (CFO)
  • Phase noise (PN)

• Classical approach: compensate impairments prior to beamforming and channel estimation

• **MmWave:** low SNR operation → classical methods may fail

• Joint wideband mmWave channel parameters, PN and CFO estimation

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System Model

Time-Domain Protocol

- Sample period $T_o$
- Symbol period $T_s$: comprises $N_o$ samples $\rightarrow T_s = N_o T_o$
- Block period $T_b$: comprises $N_s$ symbols $\rightarrow T_b = N_s T_s$
- Frame period $T_f$: comprises $N_b$ blocks $\rightarrow T_f = N_b T_b$

System Parameters

- $(N_t \times N_r)$ single-user MIMO system
- Transmission of $N_p$-length pilot sequences
- Transmit and receive codebooks of length $M_t$ and $M_r$, respectively
- Single local oscillator at each end: $\Omega$ [rad/s]
- Phase Noise: $\phi_n = \phi_{n-1} + w_n$ (Wiener process)
# System Model

## Time-Domain Protocol

- Sample period $T_o$
- Symbol period $T_s$: comprises $N_o$ samples $\rightarrow T_s = N_o T_o$
- Block period $T_b$: comprises $N_s$ symbols $\rightarrow T_b = N_s T_s$
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## System Parameters

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- Phase Noise: $\phi_n = \phi_{n-1} + w_n$ (Wiener process)
System Model

Channel Model

Channel matrix at frame $n_f$ and tap $n_c$

$$H_{n_f,n_c} = \sqrt{\frac{N_t N_r}{L}} \sum_{\ell=1}^{L} \alpha_{n_f,\ell} g_{n_c,\ell} a_r \left( \phi^{(r)}_{\ell}, \theta^{(r)}_{\ell} \right) a_t^\top \left( \phi^{(t)}_{\ell}, \theta^{(t)}_{\ell} \right) \in \mathbb{C}^{N_r \times N_t}$$

- $\alpha_{n_f,\ell}$ – frame-variant complex channel path gain
- $g_{n_c,\ell} = g(n_c T_s - \tau_\ell)$ – effective pulse shaping function

Parameters Time-Scale

- PN: Sample scale – $\phi_{n_o}, n_o = 1, \ldots, N_o$
- CFO: Block scale – $\Omega \cdot n_b, n_b = 1, \ldots, N_b$
- Channel gains: Frame scale – $\alpha_{n_f,\ell}, n_f = 1, \ldots, N_f$
System Model

Channel Model

Channel matrix at frame $n_f$ and tap $n_c$

$$H_{n_f, n_c} = \sqrt{\frac{N_t N_r}{L}} \sum_{\ell=1}^{L} \alpha_{n_f, \ell} g_{n_c, \ell} \mathbf{a}_r \left( \phi^{(r)}_{\ell}, \theta^{(r)}_{\ell} \right) \mathbf{a}_t^T \left( \phi^{(t)}_{\ell}, \theta^{(t)}_{\ell} \right) \in \mathbb{C}^{N_r \times N_t}$$

- $\alpha_{n_f, \ell}$ – frame-variant complex channel path gain
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Parameters Time-Scale

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- Channel gains: Frame scale – $\alpha_{n_f, \ell}$, $n_f = 1, \ldots, N_f$
System Model

Received signal at sample $n_o$, symbol $n_s$, block $n_b$, frame $n_f$, filtered by transmit beamformer $f_{mt}$ and receive beamformer $w_{mr}$:

$$y_{m_{r,m_{t,n_o,n_s,n_b,n_f}}^r} = e^{j(\Omega \cdot n_b + \phi_{n_o})} \sum_{n_c=0}^{N_c-1} w_{m_{r}}^H H_{n_f,n_c} f_{m_t} s_{n_s-n_c} + w_{m_{r}}^H b_{m_{t,n_b,n_f,n_s,n_o}}$$

<table>
<thead>
<tr>
<th>Frame scale (Fading)</th>
<th>$\alpha_1,\ell$</th>
<th>$\alpha_2,\ell$</th>
<th>$N_f = 2$ frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>TX BF index</td>
<td>$f_1$</td>
<td>$f_2$</td>
<td>$M_t = 2$ filters</td>
</tr>
<tr>
<td>Block scale (CFO)</td>
<td>$e^{j\Omega}$</td>
<td>$e^{j\Omega \cdot 2}$</td>
<td>$N_b = 4$ blocks</td>
</tr>
<tr>
<td>RX BF index</td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$M_r = 2$ filters</td>
</tr>
<tr>
<td>Symbol scale</td>
<td>$s_1$</td>
<td>$s_2$</td>
<td>$N_s = 8$ symbols</td>
</tr>
<tr>
<td>Sample scale (PN)</td>
<td>$e^{j\phi_1}$</td>
<td>$e^{j\phi_2}$</td>
<td>$N_o = 16$ samples</td>
</tr>
</tbody>
</table>
System Model

Received signal at sample $n_o$, symbol $n_s$, block $n_b$, frame $n_f$, filtered by transmit beamformer $f_{mt}$ and receive beamformer $w_{mr}$:

$$y_{m_r,m_t,n_o,n_s,n_b,n_f} = e^{j(\Omega \cdot n_b + \phi_{no})} \sum_{n_c=0}^{N_c-1} w^H_{m_r} H_{n_f,n_c} f_{mt} s_{n_s-n_c} + w^H_{m_r} b_{m_t,n_b,n_f,n_s,n_o}$$

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</tr>
<tr>
<td>Block scale (CFO)</td>
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<td>$e^{j\Omega \cdot 2}$</td>
<td>$e^{j\Omega \cdot 3}$</td>
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<td>$e^{j\phi_2}$</td>
<td>$e^{j\phi_3}$</td>
</tr>
</tbody>
</table>
System Model – Tensor Formulation

Effective channel tensor

\[ \mathcal{C} = \mathcal{I}_{6,L} \times_1 A_r \times_2 A_t^* \times_3 G \times_4 \Phi \times_5 \Omega \times_6 \Gamma \]  

(43)

- \( A_r \in \mathbb{C}^{N_r \times L} \) and \( A_t \in \mathbb{C}^{N_t \times L} \) – spatial signatures
- \( G \in \mathbb{C}^{N_c \times L} \) – time signature
- \( \Phi = \frac{1}{\sqrt{L}} \operatorname{Diag}(e^{j\phi_1}, \ldots, e^{j\phi_{N_o}})1_{N_o \times L} \in \mathbb{C}^{N_o \times L} \) – PN matrix (rank-1)
- \( \Omega = \frac{1}{\sqrt{L}} \operatorname{Diag}(e^{j\Omega_1}, \ldots, e^{j\Omega_{N_f}})1_{N_b \times L} \in \mathbb{C}^{N_b \times L} \) – CFO matrix
- \( \Gamma \in \mathbb{C}^{N_f \times L} \) – fading matrix

Received signal tensor

\[ \mathbf{y} = \mathcal{C} \times_1 W^H \times_2 F^T \times_3 S^T + \mathbf{z} \]  

(44)

\[ = \mathcal{I}_{6,L} \times_1 W^H A_r \times_2 F^T A_t^* \times_3 S^T G \times_4 \Phi \times_5 \Omega \times_6 \Gamma + \mathbf{z} \]  

(45)

Canonical polyadic decomposition (CPD) model!
System Model – Tensor Formulation

Effective channel tensor

\[ C = I_{6,L} \times_1 A_r \times_2 A_t^* \times_3 G \times_4 \Phi \times_5 \Omega \times_6 \Gamma \]  \tag{43}

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- \( G \in \mathbb{C}^{N_c \times L} \) – time signature

\[ \begin{aligned}
\Phi &= \frac{1}{\sqrt{L}} \text{Diag}(e^{j\phi_1}, \ldots, e^{j\phi_{N_o}})1_{N_o \times L} \in \mathbb{C}^{N_o \times L} \quad \text{– PN matrix} \\
\Omega &= \frac{1}{\sqrt{L}} \text{Diag}(e^{j\Omega}, \ldots, e^{j\Omega \cdot N_f})1_{N_b \times L} \in \mathbb{C}^{N_b \times L} \quad \text{– CFO matrix} \\
\Gamma &\in \mathbb{C}^{N_f \times L} \quad \text{– fading matrix}
\end{aligned} \]

Received signal tensor

\[ \begin{aligned}
Y &= C \times_1 W^H \times_2 F^T \times_3 S^T + Z \\
&= I_{6,L} \times_1 W^H A_r \times_2 F^T A_t^* \times_3 S^T G \times_4 \Phi \times_5 \Omega \times_6 \Gamma + Z. \tag{45}
\end{aligned} \]
System Model – Tensor Formulation

Effective channel tensor

\[ \mathcal{C} = I_{6,L} \times_1 A_r \times_2 A_t^* \times_3 G \times_4 \Phi \times_5 \Omega \times_6 \Gamma \]  \hspace{1cm} (43)

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  \hspace{1cm} (rank-1)
- \( \Omega = \frac{1}{\sqrt{L}} \text{Diag}(e^{j\Omega}, \ldots, e^{j\Omega \cdot N_f})1_{N_b \times L} \in \mathbb{C}^{N_b \times L} \) – CFO matrix
- \( \Gamma \in \mathbb{C}^{N_f \times L} \) – fading matrix

Received signal tensor

\[ \mathcal{Y} = \mathcal{C} \times_1 W^H \times_2 F^T \times_3 S^T + \mathcal{Z} \hspace{1cm} (44) \]
\[ = I_{6,L} \times_1 W^H A_r \times_2 F^T A_t^* \times_3 S^T G \times_4 \Phi \times_5 \Omega \times_6 \Gamma + \mathcal{Z}. \hspace{1cm} (45) \]

Canonical polyadic decomposition (CPD) model!
System Model – Tensor Formulation

Effective channel tensor

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System Model – Tensor Formulation

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System Model – Tensor Formulation

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System Model – Tensor Formulation

Effective channel tensor

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\[ Y = C \times_1 W^H \times_2 F^T \times_3 S^T + Z \]

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System Model – Tensor Formulation

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Canonical polyadic decomposition (CPD) model!
Parameter Estimation

\[ \mathcal{Y} = I_{6,L} \times_1 W^H A_r \times_2 F^T A_t^* \times_3 S^T G \times_4 \Phi \times_5 \Omega \times_6 \Gamma + \mathcal{Z} \]

Steps

1. Factorize received signal tensor \( \mathcal{Y} \) into CPD model
2. Solve permutation ambiguity
3. Estimate the path angles and delays by solving sparse recovery problems

\[
\begin{align*}
\min_{\nu_r} & \quad \| \nu_r \|_1 \\
\text{subject to} & \quad \| q(1) - [I_L \otimes (W^H \Psi_r)] \nu_r \|_2 \leq \sigma, \\
\min_{\nu_t} & \quad \| \nu_t \|_1 \\
\text{subject to} & \quad \| q(2) - [I_L \otimes (F^T \Psi_t)] \nu_t \|_2 \leq \sigma, \\
\min_{\nu_s} & \quad \| \nu_s \|_1 \\
\text{subject to} & \quad \| q(3) - [I_L \otimes (S^T \Psi_s)] \nu_s \|_2 \leq \sigma,
\end{align*}
\]

4. Estimate PN and CFO directly from the CPD factors
5. Estimate channel fading matrix as

\[ \hat{\Gamma} = Y(6) \left\{ \left[ \Omega \diamond \Phi \diamond (S^T G) \diamond (F^T A_t^*) \diamond (W^H A_r) \right]^T \right\}^\dagger. \]
Parameter Estimation

\[ \mathcal{Y} = \mathcal{I}_{6, L} \times_1 W^H A_r \times_2 F^T A_t^* \times_3 S^T G \times_4 \Phi \times_5 \Omega \times_6 \Gamma + \mathcal{Z} \]

**Steps**

1. Factorize received signal tensor \( \mathcal{Y} \) into CPD model
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\begin{align*}
\text{minimize} & \quad ||\nu_r||_1 \\
\text{subject to} & \quad ||q(1) - [I_L \otimes (W^H \Psi_r)]\nu_r||_2 \leq \sigma, \\
\text{minimize} & \quad ||\nu_t||_1 \\
\text{subject to} & \quad ||q(2) - [I_L \otimes (F^T \Psi_t)]\nu_t||_2 \leq \sigma, \\
\text{minimize} & \quad ||\nu_s||_1 \\
\text{subject to} & \quad ||q(3) - [I_L \otimes (S^T \Psi_s)]\nu_s||_2 \leq \sigma,
\end{align*}
\]

(46) (47) (48)

4. Estimate PN and CFO directly from the CPD factors
5. Estimate channel fading matrix as

\[
\hat{\Gamma} = \mathcal{Y}(6) \left\{ [\Omega \circ \Phi \circ (S^T G) \circ (F^T A_t^*) \circ (W^H A_r)]^T \right\}^\dagger.
\]

(49)
Parameter Estimation

\[ \mathcal{Y} = I_{6,L} \times_1 W^H A_r \times_2 F^T A_t^* \times_3 S^T G \times_4 \Phi \times_5 \Omega \times_6 \Gamma + \mathcal{Z} \]

Steps

1. Factorize received signal tensor \( \mathcal{Y} \) into CPD model
2. Solve permutation ambiguity
3. Estimate the path angles and delays by solving sparse recovery problems

\[
\begin{align*}
\text{minimize} & \quad \| \nu_r \|_1 \\
\text{subject to} & \quad \| q(1) - [I_L \otimes (W^H \Psi_r)] \nu_r \|_2 \leq \sigma,
\end{align*}
\]

(46)

\[
\begin{align*}
\text{minimize} & \quad \| \nu_t \|_1 \\
\text{subject to} & \quad \| q(2) - [I_L \otimes (F^T \Psi_t)] \nu_t \|_2 \leq \sigma,
\end{align*}
\]

(47)

\[
\begin{align*}
\text{minimize} & \quad \| \nu_s \|_1 \\
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(48)

4. Estimate PN and CFO directly from the CPD factors
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\[
\hat{\Gamma} = Y_{(6)} \left\{ \left[ \Omega \diamond \Phi \diamond (S^T G) \diamond (F^T A_t^*) \diamond (W^H A_r) \right]^T \right\}^\dagger.
\]

(49)
Parameter Estimation

\[ \mathcal{Y} = I_{6,L} \times 1 \ W^H A_r \times 2 \ F^T A_t^* \times 3 \ S^T G \times 4 \ \Phi \times 5 \ \Omega \times 6 \ \Gamma + \mathcal{Z} \]

**Steps**

1. Factorize received signal tensor \( \mathcal{Y} \) into CPD model
2. Solve permutation ambiguity
3. Estimate the path angles and delays by solving sparse recovery problems

\[
\begin{align*}
\text{minimize} & \quad \|\nu_r\|_1 \\
\text{subject to} & \quad \|q(1) - [I_L \otimes (W^H \Psi_r)]\nu_r\|_2 \leq \sigma, \\
\text{minimize} & \quad \|\nu_t\|_1 \\
\text{subject to} & \quad \|q(2) - [I_L \otimes (F^T \Psi_t)]\nu_t\|_2 \leq \sigma, \\
\text{minimize} & \quad \|\nu_s\|_1 \\
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\end{align*}
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\]
Parameter Estimation

\[ \mathcal{Y} = \mathcal{I}_{6,L} \times_1 W^H A_r \times_2 F^T A_t^* \times_3 S^T G \times_4 \Phi \times_5 \Omega \times_6 \Gamma + \mathcal{Z} \]

**Steps**

1. Factorize received signal tensor \( \mathcal{Y} \) into CPD model
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\begin{align*}
\text{minimize} & \quad \| \nu_r \|_1 \\
\text{subject to} & \quad \| q_{(1)} - [I_L \otimes (W^H \Psi_r)] \nu_r \|_2 \leq \sigma, \\
\text{minimize} & \quad \| \nu_t \|_1 \\
\text{subject to} & \quad \| q_{(2)} - [I_L \otimes (F^T \Psi_t)] \nu_t \|_2 \leq \sigma, \\
\text{minimize} & \quad \| \nu_s \|_1 \\
\text{subject to} & \quad \| q_{(3)} - [I_L \otimes (S^T \Psi_s)] \nu_s \|_2 \leq \sigma,
\end{align*}
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\hat{\Gamma} = Y(6) \left\{ \left[ \Omega \diamond \Phi \diamond (S^T G) \diamond (F^T A_t^*) \diamond (W^H A_r) \right]^T \right\}^\dagger.
\]
Simulation Results

Figures of Merit

Angles, delays and CFO ($L = 1$)

\[ \text{NMSE}(x) = \sum_{\ell=1}^{L} \frac{|x_\ell - \hat{x}_\ell|^2}{|x_\ell|^2} \]  
(50)

Phase noise

\[ \text{NMSE}(\phi) = \sum_{n_o=1}^{N_o} \frac{|\phi_{n_o} - \hat{\phi}_{n_o}|^2}{|\phi_{n_o}|^2} \]  
(51)

Fading matrix

\[ \text{NMSE}(\mathbf{\Gamma}) = \frac{||\mathbf{\Gamma} - \hat{\mathbf{\Gamma}}||_F^2}{||\mathbf{\Gamma}||_F^2} \]  
(52)

Calculate NMSE for different codebook lengths and samples number $N_o$

Parameter Setup

- $N_t = N_r = 16$ antennas
- $N_s = N_b = N_f = 2$
- Sampling period $T_s = 0.1 \, \mu s$
- Carrier frequency 28 GHz
- 10 ppm CFO: 280 kHz
- 2000 independent trials
Simulation Results

Figures of Merit

Angles, delays and CFO \((L = 1)\)

\[
\text{NMSE}(x) = \sum_{\ell=1}^{L} \frac{|x_\ell - \hat{x}_\ell|^2}{|x_\ell|^2} \tag{50}
\]

Phase noise

\[
\text{NMSE}(\phi) = \sum_{n_o=1}^{N_o} \frac{|\phi_{n_o} - \hat{\phi}_{n_o}|^2}{|\phi_{n_o}|^2} \tag{51}
\]

Fading matrix

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Angles of arrival

Path delays
Simulation Results

Angles of arrival

Path delays

SNR [dB]

NMSE

$N_o = 16$ samples, $M_t = M_r = 8$
$N_o = 16$ samples, $M_t = M_r = 16$
$N_o = 32$ samples, $M_t = M_r = 8$
$N_o = 32$ samples, $M_t = M_r = 16$
Simulation Results

![Graphs showing simulation results for SNR in dB and NMSE]
Simulation Results

Angles of arrival

Path delays
Simulation Results

Angles of arrival

Path delays
Simulation Results

Angles of arrival

Path delays
Simulation Results

Angles of arrival

Path delays
Simulation Results

SNR [dB] vs NMSE for different numbers of samples and modem rates. The plots show the performance of the system under varying conditions of angles of arrival and path delays.
Simulation Results

Phase noise

CFO

$N_0 = 16$ samples, $M_t = M_r = 8$
$N_0 = 16$ samples, $M_t = M_r = 16$
$N_0 = 32$ samples, $M_t = M_r = 8$
$N_0 = 32$ samples, $M_t = M_r = 16$
Simulation Results

**Phase noise**

- $N_o = 16$ samples, $M_t = M_r = 8$
- $N_o = 16$ samples, $M_t = M_r = 16$
- $N_o = 32$ samples, $M_t = M_r = 8$
- $N_o = 32$ samples, $M_t = M_r = 16$

**CFO**

- $N_o = 16$ samples, $M_t = M_r = 8$
- $N_o = 16$ samples, $M_t = M_r = 16$
- $N_o = 32$ samples, $M_t = M_r = 8$
- $N_o = 32$ samples, $M_t = M_r = 16$
Simulation Results

![Graph showing NMSE vs SNR with varying parameters](image)

- $N_0 = 16$ samples, $M_t = M_r = 8$
- $N_0 = 16$ samples, $M_t = M_r = 16$
- $N_0 = 32$ samples, $M_t = M_r = 8$
- $N_0 = 32$ samples, $M_t = M_r = 16$

**Phase noise**

**CFO**
Simulation Results

- SNR [dB]
- NMSE
- Phase noise
- CFO

Different scenarios are shown with varying numbers of samples and antenna configurations:
- $N_o = 16$ samples, $M_t = M_r = 8$
- $N_o = 16$ samples, $M_t = M_r = 16$
- $N_o = 32$ samples, $M_t = M_r = 8$
- $N_o = 32$ samples, $M_t = M_r = 16$
Simulation Results

![Graph](image)

Phase noise

CFO
Simulation Results

Phase noise

CFO
Simulation Results

For phase noise and CFO, the plots show the NMSE as a function of SNR [dB]. The graphs illustrate the relationship between the number of samples and the transmit and receive antenna configurations. The plots are labeled with different configurations, such as $N_o = 16$ samples, $M_t = M_r = 8$, and $N_o = 32$ samples, $M_t = M_r = 16$. These configurations help in understanding how different system parameters affect the performance under varying SNR conditions.
Simulation Results

Phase noise

CFO

SNR [dB]

NMSE

$N_o = 16$ samples, $M_t = M_r = 8$

$N_o = 16$ samples, $M_t = M_r = 16$

$N_o = 32$ samples, $M_t = M_r = 8$

$N_o = 32$ samples, $M_t = M_r = 16$
Conclusion

Part I
- Low-complexity tensor beamforming filters
- Low-rank extension of tensor filters

Part II
- Energy efficiency analysis of precoding structures for mmWave massive MIMO
- Double-sided massive MIMO transceiver schemes

Part III
- Tensor methods for joint wideband channel parameters, phase noise and CFO
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Some Perspectives

- Tensor filters extensions
  - Tensor train model
  - Unsupervised strategies
- Wide-band and multi-carrier extensions of the proposed massive MIMO methods
- Transceiver performance under imperfect CSI
- Channel parameter tracking
Some Perspectives

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## Publications

### Journal Papers

1. IEEE JSTSP 2018 – *Energy efficiency of mmWave massive MIMO precoding with low-resolution DACs*
2. Signal Processing 2019 – *Separable linearly constrained minimum variance beamformers*
3. IET Signal Processing 2019 – *Low-complexity separable beamformers for massive antenna array systems*
4. Under revision – *Double-sided massive MIMO transceivers for mmWave communications*
Publications

Conference Papers

1. EUSIPCO’17 – A low-complexity equalizer for massive MIMO systems based on array separability
2. SBRT’18 – Separable least-mean squares beamforming
3. ICASSP’19 – Tensor-based estimation of mmWave MIMO channels with carrier frequency offset
4. ISWCS’19 – Low-rank tensor MMSE equalization
5. Under preparation – Joint phase noise and carrier frequency offset estimation in wideband mmWave MIMO channels
Doctoral Thesis Defense

Signal Processing Methods for Large-Scale Multi-Antenna Systems

Lucas Nogueira Ribeiro

Advisor: Prof. Dr. André Lima Férrer de Almeida
Co-Advisor: Prof. Dr. João César Moura Mota

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