TENSOR BEAMFORMING FOR MULTILINEAR TRANSLATION INVARIANT ARRAYS

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ABSTRACT

In the past few years, multidimensional array processing emerged as the generalization of classic array signal processing. Tensor methods exploiting array multidimensionality provided more accurate parameter estimation and consistent modeling. In this paper, multilinear translation invariant arrays are studied. An M-dimensional translation invariant array admits a separable representation in terms of a reference subarray and a set of $M-1$ translations, which is equivalent to a rank-1 decomposition of an Mth order array manifold tensor. We show that such a multilinear translation invariant property can be exploited to design tensor beamformers that operate multilinearly on the subarray level instead of the global array level, which is usually the case with a linear beamforming. An important reduction of the computational complexity is achieved with the proposed tensor beamformer with a negligible loss in performance compared to the classical minimum mean square error (MMSE) beamforming solution.

***Index Terms*—** Array processing, beamforming, tensor filtering.

1. INTRODUCTION

Array signal processing techniques have been used in the last decades in several area of applications such as: communications systems [1], audio processing [2], biomedical engineering [3], among others. An array consists of multiple sensors placed in different locations in space to process the impinging signals using a spatial filter. This filter is a *beamformer* when it is employed to enhance a signal of interest (SOI) arriving from a certain direction while attenuating any possible interfering signal [4].

In the past few years, generalized models for array processing have been proposed for taking advantage of the multidimensionality present in many types of arrays [1, 5, 6, 7]. For instance, in [8] a multidimensional harmonic retrieval method that improved the parameter estimation accuracy was proposed. Model selection methods for such multidimensional models were proposed in [9]. In [5], the authors introduced the concept of translation invariant arrays and proposed a joint channel and source estimation method based on the coherence properties of the sources. By contrast, very few works have concentrated on multidimensional beamforming. In [10], the authors proposed a MVDR-based beamformer that relies on the PARAFAC decomposition to estimate the DOA of the SOI. Recently, a multidimensional generalized sidelobe canceller (GSC) beamformer has been proposed in [11]. The separability of a uniform rectangular array was exploited by the proposed technique, resulting in better SOI estimation and reduced computational complexity compared to the classic GSC. Both tensor beamformers rely on a prior DOA estimation stage, which is then used to derive the filter coefficients. In [12], the authors exploited the separability of the impulse response of a linear time-invariant system and proposed a trilinear filtering system identification algorithm based on a tensor approach. Therein, it is shown that the tensor approach provides a more accurate system identification with a reduced computational complexity compared to its linear counterpart that ignores system separability property.

In this paper, we first extend the translation invariance property presented in [5] to multiple translation vectors. More specifically, we start from an M-dimensional translation invariant array that admits a separable representation in terms of a reference subarray and a set of $M-1$ translations, which is equivalent to a rank-1 decomposition of an Mth order array manifold tensor. We show that such a multilinear translation invariant property can be exploited to design tensor beamformers that operate multilinearly on the subarray level instead of operating linearly on the global array level, which is the case with classical beamformers. Hence, an important reduction of the computational complexity can be achieved by the tensor beamformer, with a negligible loss in performance compared to the conventional linear minimum mean square error (MMSE) beamforming. According to our numerical results, the number of FLOPS demanded by the proposed method is remarkably lower than that of the linear (vector-based) MMSE filter for $M=3,4$ even though their SOI estimation quality are essentially the same. Moreover, since the separability degrees of freedom increase with the number of sensors in the multidimensional array, the tensor beamforming approach is particularly interesting for large-scale (massive) sensor arrays.

1.1. Notation

Scalars are denoted by lowercase letters, vectors by lowercase boldface letters, matrices by uppercase boldface letters, and higher-order tensors by calligraphic letters. The Kronecker, outer, and n-mode products are denoted by the symbols $\otimes$, $\circ$, and $\times_n$, respectively. The ℓ^2 norm, statistical expectation, inner product, and n-mode tensor concatenation are denoted by $\|\cdot\|_2^2$, $\mathbb{E}[\cdot]$, $\langle\cdot,\cdot\rangle$, and $\sqcup_n$, respectively. The transpose and Hermitian operators are denoted by $(\cdot)^{\mathsf{T}}$ and $(\cdot)^{\mathsf{H}}$, respectively.

2. MULTILINEAR TRANSLATION INVARIANT ARRAYS

In this section, arrays enjoying the translation invariance property will be studied. Then, a tensor beamforming approach exploiting the multilinearity present in the translation invariant arrays will be formulated. First, let us review some tensor prerequisites for convenience.

2.1. Tensor prerequisites

In this work, an Nth order tensor is defined as an N-dimensional array. For instance, $\mathcal{T} \in \mathbb{C}^{I_1 \times I_2 \times \ldots \times I_N}$ is an Nth order tensor whose elements are denoted by $t_{i_1,i_2,\ldots,i_N} = [\mathcal{T}]_{i_1,i_2,\ldots,i_N}$ where $i_n \in \{1,\ldots,I_N\}$, $n=1,2,\ldots,N$.

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The $\{1, \ldots, N\}$-*mode products* of $\mathcal{T}$ with N matrices $\{\mathbf{U}^{(n)}\}_{n=1}^{N}$ yield the tensor $\tilde{\mathcal{T}} = \mathcal{T} \times_1 \mathbf{U}^{(1)} \ldots \times_N \mathbf{U}^{(N)} \in \mathbb{C}^{J_1 \times \ldots \times J_N}$ defined as [13]

$$[\tilde{\mathcal{T}}]_{j_1,\ldots,j_N} = \sum_{i_1=1}^{I_1} \cdots \sum_{i_N=1}^{I_N} t_{i_1,\ldots,i_N} u^{(1)}_{j_1,i_1} \ldots u^{(N)}_{j_N,i_N},$$

where $\mathbf{U}^{(n)} \in \mathbb{C}^{J_n \times I_n}$, $i_n \in \{1, \ldots, I_n\}$, and $j_n \in \{1, \ldots, J_n\}$, $n = 1, \ldots, N$. The n-mode unfolding of $\tilde{\mathcal{T}}$ is given by

$$\tilde{\mathbf{T}}_{(n)} = \mathbf{U}^{(n)} \mathbf{T}_{(n)} \mathbf{U}^{\otimes n^{\mathsf{T}}}, \tag{1}$$

where $\mathbf{T}_{(n)}$ denotes the n-mode unfolding of $\mathcal{T}$, and

$$\mathbf{U}^{\otimes n} = \mathbf{U}^{(N)} \otimes \ldots \otimes \mathbf{U}^{(n+1)} \otimes \mathbf{U}^{(n-1)} \otimes \ldots \otimes \mathbf{U}^{(1)} \tag{2}$$

denotes the Kronecker product of the matrices $\{\mathbf{U}^{(j)}\}_{j=1,j\neq n}^{N}$ in the decreasing order. Note that the $\{1, \ldots, N\}$-mode products of $\mathcal{T}$ with the N vectors $\{\mathbf{u}^{(n)}\}_{n=1}^{N}$ yields a scalar $t = \mathcal{T} \times_1 \mathbf{u}^{(1)^{\mathsf{T}}} \ldots \times_N \mathbf{u}^{(N)^{\mathsf{T}}}$ where $\mathbf{u}^{(n)} \in \mathbb{C}^{I_n \times 1}$, $n = 1, \ldots, N$.

The *inner product* between $\mathcal{A}, \mathcal{B} \in \mathbb{C}^{I_1 \times \ldots \times I_N}$ is defined as

$$\langle \mathcal{A}, \mathcal{B} \rangle = \sum_{i_1=1}^{I_1} \cdots \sum_{i_N=1}^{I_N} a_{i_1,\ldots,i_N} b_{i_1,\ldots,i_N},$$

where $a_{i_1,\ldots,i_N} = [\mathcal{A}]_{i_1,\ldots,i_N}$ and $b_{i_1,\ldots,i_N} = [\mathcal{B}]_{i_1,\ldots,i_N}$.

The *tensorization* operator $\Theta : \mathbb{C}^{\prod_{n=1}^{N} I_n} \to \mathbb{R}^{I_1 \times \ldots \times I_N}$ is defined as $[\Theta(\mathbf{v})]_{i_1,\ldots,i_N} = [\mathbf{v}]_j$, where $\mathbf{v} \in \mathbb{C}^{\prod_{n=1}^{N} I_n}$ is an input vector, and $j = i_1 + \sum_{\mu=2}^{M} (i_\mu - 1) \prod_{v=1}^{\mu-1} I_v$ for $i_\mu \in \{1, \ldots, I_\mu\}$.

2.2. Signal model

Consider a sensor array composed of N isotropic sensors located at $\tilde{\mathbf{p}}_n \in \mathbb{R}^{3 \times 1}$ for $n = 1, \ldots, N$. This array will be hereafter referred to as the *global array*. Consider that R narrowband source signals with complex amplitudes $s_r(k)$ impinge on the global array from directions $\mathbf{d}_r = [\sin\theta_r \cos\phi_r, \sin\theta_r \sin\phi_r, \cos\theta_r]^{\mathsf{T}}$, where θ_r and ϕ_r denote the elevation and azimuth angles, respectively, $r = 1, \ldots, R$. The sources are assumed to be in the far-field and it is assumed that there are no reflection components. The steering vector associated with the rth source is given by

$$\mathbf{a}(\mathbf{d}_r) = \left[e^{\jmath \frac{\omega}{c} \tilde{\mathbf{p}}_1^{\mathsf{T}} \mathbf{d}_r}, \ldots, e^{\jmath \frac{\omega}{c} \tilde{\mathbf{p}}_N^{\mathsf{T}} \mathbf{d}_r} \right]^{\mathsf{T}} \in \mathbb{C}^{N \times 1}, \tag{3}$$

where $\jmath = \sqrt{-1}$, c denotes the velocity of propagation in the medium, and ω is the wave frequency. The signals collected by the global array at instant k are modeled as

$$\mathbf{x}(k) = \sum_{r=1}^{R} \mathbf{a}(\mathbf{d}_r) s_r(k) + \mathbf{b}(k) \in \mathbb{C}^{N \times 1}, \tag{4}$$

where $\mathbf{b}(k) \in \mathbb{C}^{N \times 1}$ is the additive zero-mean complex white Gaussian noise vector with covariance matrix equal to $\sigma^2 \mathbf{I}$.

Lim and Comon presented in [5, 14] the concept of translation invariant arrays formed by translating a *reference subarray*. However, the model discussed therein was limited to one translation vector only. This idea is now generalized to multiple translation vectors, leading to a multilinear array structure that will be useful in our context.

Let us assume that the global array enjoys the multilinear translation invariance property. Consider a reference subarray formed by N_1 reference sensors located at $\mathbf{p}^{(1)}_{n_1}$ for $n_1 = 1, \ldots, N_1$. The n_1th reference sensor is translated $M - 1$ times by means of the translation vectors $\mathbf{p}^{(2)}_{n_2}, \ldots, \mathbf{p}^{(M)}_{n_M}$, yielding the following decomposition for the nth global sensor location vector

$$\tilde{\mathbf{p}}_n = \mathbf{p}^{(1)}_{n_1} + \mathbf{p}^{(2)}_{n_2} + \ldots + \mathbf{p}^{(M)}_{n_M}, \tag{5}$$

where $n = n_1 + \sum_{\mu=2}^{M} (n_\mu - 1) \prod_{v=1}^{\mu-1} N_v$, $n_\mu \in \{1, \ldots, N_\mu\}$. Note that $m = 1$ refers to the reference subarray, whereas $2 \leq m \leq M$ refers to the mth order translation vector. Substituting (5) into the global array vector (3) leads to the following separable form:

$$\begin{aligned} \mathbf{a}(\mathbf{d}_r) &= \begin{bmatrix} e^{\jmath \frac{\omega}{c} \mathbf{p}_1^{(1)^{\mathsf{T}}} \mathbf{d}_r} \ldots e^{\jmath \frac{\omega}{c} \mathbf{p}_1^{(M)^{\mathsf{T}}} \mathbf{d}_r} \\ \vdots \\ e^{\jmath \frac{\omega}{c} \mathbf{p}_{N_1}^{(1)^{\mathsf{T}}} \mathbf{d}_r} \ldots e^{\jmath \frac{\omega}{c} \mathbf{p}_{N_M}^{(M)^{\mathsf{T}}} \mathbf{d}_r} \end{bmatrix} \\ &= \mathbf{a}^{(1)}(\mathbf{d}_r) \otimes \ldots \otimes \mathbf{a}^{(M)}(\mathbf{d}_r) \in \mathbb{C}^{\prod_{m=1}^{M} N_m}, \end{aligned} \tag{6}$$

where $\mathbf{a}^{(m)}(\mathbf{d}_r) = [e^{\jmath \frac{\omega}{c} \mathbf{p}_1^{(m)^{\mathsf{T}}} \mathbf{d}_r}, \ldots, e^{\jmath \frac{\omega}{c} \mathbf{p}_{N_m}^{(m)^{\mathsf{T}}} \mathbf{d}_r}]^{\mathsf{T}} \in \mathbb{C}^{N_m \times 1}$ denotes the subarray vector associated with the mth order translation vector $\mathbf{p}^{(m)}_{n_m}$, $n_m \in \{1, \ldots, N_m\}$. A close idea was presented in [7] (therein referred to as multi-scale arrays), although the translation structure and its interpretation are different from the one we consider in this paper. Indeed, (6) is a vectorization of a rank-1 array steering tensor defined as

$$\mathcal{A}(\mathbf{d}_r) = \mathbf{a}^{(M)}(\mathbf{d}_r) \circ \ldots \circ \mathbf{a}^{(1)}(\mathbf{d}_r) \in {}^{N_1 \times \ldots \times N_M}. \tag{7}$$

In view of this, the received signals (4) can be expressed as a linear combination of R rank-1 tensors:

$$\mathcal{X}(k) = \sum_{r=1}^{R} \mathcal{A}(\mathbf{d}_r) s_r(k) + \mathcal{B}(k), \tag{8}$$

where $\mathcal{B}(k) = \Theta(\mathbf{b}(k)) \in \mathbb{C}^{N_1 \times \ldots \times N_M}$ is the tensorized form of the noise vector $\mathbf{b}(k)$.

The multilinearity inherent to translation invariant arrays allows us to decompose the array response into multiple setups, as illustrated in Fig. 1. From this figure, it can be seen that the same 3-D global array can be decomposed as two separable (3-D and 1-D) subarrays ($M = 1$ translations), or three separable (2-D, 1-D, and 1-D) subarrays ($M = 2$ translations), or as four 1-D separable arrays ($M = 3$ translations). Other decompositions are possible, and the number of possibilities increases as a function of the number of sensors in the global array.

In the following, we exploit the multilinear translation invariant property to design tensor beamformers that operate on the subarray level instead of the global array level. By adopting a multilinear structure for the beamforming filters, we can obtain a considerable reduction on the computational complexity of the spatial filtering, with almost no loss in performance, as will be clear in later sections.

3. TENSOR BEAMFORMING

Classical linear beamforming methods [4] based on model (4) ignore the multilinearity that may be present in translation invariant arrays. For convenience, in this work we consider the MMSE beamforming solution, although the proposed approach is also applicable to other beamforming solutions. The well-known solution for the MMSE beamforming problem is given by

$$\mathbf{w}_{\text{MMSE}} = \mathbf{R}_x^{-1} \mathbf{p}_{d\mathbf{x}}, \tag{9}$$

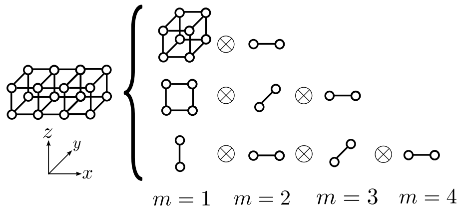

Fig. 1. An $4 \times 2 \times 2$ volumetric array decomposed in three different forms, in terms of subarrays translations ($M = 2, 3$, and 4). Reference subarrays are indexed by $m = 1$, whereas $m > 1$ refers to translation.

where $\mathbf{R}_x = \mathbb{E}\left[\mathbf{x}(k)\mathbf{x}(k)^{\mathsf{H}}\right] \in \mathbb{C}^{N \times N}$ is the autocorrelation matrix of the received vector, $\mathbf{p}_{d\mathbf{x}} = \mathbb{E}\left[s^*_{\text{SOI}}(k)\mathbf{x}(k)\right]$ is the cross-correlation vector between the received vector and the SOI. Since the MMSE filter depends on the inversion of an $N \times N$ matrix, the computational cost of (9) is $\mathcal{O}(N^2)$. Although the computational cost is not an issue for small sensor arrays, it may become prohibitively expensive for large-scale (massive) arrays of sensors. However, if the global array is translation invariant, then each mth subarray vector could be estimated/filtered in a lower-dimensional space, conditioned on the other subarray vectors thanks to the separability property of the array structure. Hereafter, instead of designing the beamforming coefficients from the spatial signatures or from prior DOA estimates (the common approach), we propose a direct beamforming method that exploits the array separability property in (8).

Let us consider an Mth order tensor filter $\mathcal{W} \in \mathbb{C}^{N_1 \times N_2 \times \ldots \times N_M}$, each mode of which is associated with a different subarray. The output of the tensor beamformer is given by

$$y(k) = \langle \mathcal{X}(k), \mathcal{W}^* \rangle. \tag{10}$$

The tensor beamformer $\mathcal{W}$ can be designed to minimize the following cost function

$$J(\mathcal{W}) = \mathbb{E}\left[|s_{\text{SOI}}(k) - \langle \mathcal{X}(k), \mathcal{W}^* \rangle|^2\right], \tag{11}$$

defined as the MSE between $y(k)$ and the SOI $s_{\text{SOI}}(k)$. We assume that the tensor filter is rank-1, i.e. $\mathcal{W} = \mathbf{w}_1 \circ \ldots \circ \mathbf{w}_M$, where $\mathbf{w}_m \in \mathbb{C}^{N_m \times 1}$ for $m \in \{1, \ldots, M\}$. In this case, Eq. (10) becomes $\{1, \ldots, M\}$-mode products between $\mathcal{X}(k)$ and $\{\mathbf{w}^*_n\}_{m=1}^M$:

$$\begin{aligned} y(k) &= \sum_{n_1=1}^{N_1} \cdots \sum_{n_M=1}^{N_M} [\mathcal{X}(k)]_{n_1,\ldots,n_M} [\mathcal{W}]^*_{n_1,\ldots,n_M} \\ &= \sum_{n_1=1}^{N_1} \cdots \sum_{n_M=1}^{N_M} [\mathcal{X}(k)]_{n_1,\ldots,n_M} [\mathbf{w}_1]^*_{n_1} \ldots [\mathbf{w}_M]^*_{n_M} \\ &= \mathcal{X}(k) \times_1 \mathbf{w}_1^{\mathsf{H}} \ldots \times_M \mathbf{w}_M^{\mathsf{H}}. \end{aligned} \tag{12}$$

Substituting (8) into (12), ignoring the noise component for simplicity, and applying the $\{1, \ldots, M\}$-mode products yields the following output signal

$$y(k) = \sum_{r=1}^{R} \left[\mathbf{w}_1^{\mathsf{H}} \mathbf{a}^{(1)}(\mathbf{d}_r)\right] \circ \ldots \circ \left[\mathbf{w}_M^{\mathsf{H}} \mathbf{a}^{(M)}(\mathbf{d}_r)\right] s_r(k). \tag{13}$$

Equation (13) shows that the multilinearity imposed on the beamforming tensor $\mathcal{W}$ exploits the separability property of the translation invariant array, i.e., by processing each dimension of $\mathcal{X}(k)$ separately. Due to multilinearity of the tensor beamforming, the cost function $J(\mathcal{W})$ can be rewritten in M equivalent forms, with respect to each subfilter:

$$\begin{aligned} J(\mathcal{W}) &= \mathbb{E}\left[|s_{\text{SOI}}(k) - \mathcal{X}(k) \times_1 \mathbf{w}_1^{\mathsf{H}} \ldots \times_M \mathbf{w}_M^{\mathsf{H}}|^2\right] && (14) \\ &= \mathbb{E}\left[|s_{\text{SOI}}(k) - \mathbf{w}_m^{\mathsf{H}} \mathbf{X}_{(m)}(k) \left[\mathbf{w}^{\otimes m}\right]^* |^2\right] && (15) \\ &= \mathbb{E}\left[|s_{\text{SOI}}(k) - \mathbf{w}_m^{\mathsf{H}} \mathbf{u}_m(k)|^2\right], && (16) \end{aligned}$$

where $\mathbf{u}_m(k) = \mathbf{X}_{(m)}(k)\left[\mathbf{w}^{\otimes m}\right]^* \in \mathbb{C}^{I_m \times 1}$ for $m = 1, \ldots, M$, and $\mathbf{w}^{\otimes m}$ is defined analogously to (2). Note that the n-mode unfolding (1) is used in (14) to obtain (15). Deriving (16) with respect to $\mathbf{w}^*_m$ and equating the result to $\mathbf{0} \in \mathbb{C}^{N_m \times 1}$ yields:

$$\frac{\partial J(\mathcal{W})}{\partial \mathbf{w}^*_m} = -\mathbf{p}_m + \mathbf{R}_m \mathbf{w}_m = \mathbf{0} \Rightarrow \hat{\mathbf{w}}_m = \mathbf{R}_m^{-1} \mathbf{p}_m, \tag{17}$$

where $\mathbf{p}_m = \mathbb{E}\left[\mathbf{u}_m(k) s^*_{\text{SOI}}(k)\right] \in \mathbb{C}^{N_m \times 1}$ is the cross-correlation vector between $\mathbf{u}_m(k)$ and $s_{\text{SOI}}(k)$, and $\mathbf{R}_m = \mathbb{E}\left[\mathbf{u}_m(k)\mathbf{u}_m(k)^{\mathsf{H}}\right] \in \mathbb{C}^{N_m \times N_m}$ is the autocorrelation matrix associated with the mth subarray.

Multilinear MMSE beamforming

Standard optimization methods do not guarantee global convergence when minimizing (14) due to its joint nonconvexity with respect to all the variables. The alternating minimization approach [12, 15] has demonstrated to be a solution to solve the global nonlinear problem in terms of M smaller linear problems. It consists in updating the mth mode beamforming filter each time by solving (17) for $\mathbf{w}_m$, while $\{\mathbf{w}_j\}_{j=1, j\neq m}^M$ remain fixed, $m = 1, \ldots, M$, conditioned on the previous updates of the other filters.

Define $\mathcal{X} = [\mathcal{X}(k) \sqcup_{M+1} \ldots \sqcup_{M+1} \mathcal{X}(k - K + 1)] \in \mathbb{C}^{N_1 \times \ldots \times N_M \times K}$ as the concatenation of K time snapshots of $\mathcal{X}(k)$ along the $(M+1)$th dimension. Let $\mathbf{U}^{(m)} \in \mathbb{C}^{N_m \times K}$ denote the $\{1, \ldots, m-1, m+1, \ldots, N\}$-mode products between $\mathcal{X}$ and $\{\mathbf{w}_j\}_{j=1, j\neq m}^M$:

$$\mathbf{U}^{(m)} = \mathcal{X} \times_1 \mathbf{w}_1^{\mathsf{H}} \ldots \times_{m-1} \mathbf{w}_{m-1}^{\mathsf{H}} \times_{m+1} \mathbf{w}_{m+1}^{\mathsf{H}} \ldots \times_M \mathbf{w}_M^{\mathsf{H}}. \tag{18}$$

It can be shown that $\mathbf{U}^{(m)} = [\mathbf{u}_m(k), \ldots, \mathbf{u}_m(k - K + 1)]$. Therefore the sample estimate of $\mathbf{R}_n$ and $\mathbf{p}_n$ are given by:

$$\hat{\mathbf{R}}_n = \frac{1}{K} \mathbf{U}^{(m)} \mathbf{U}^{(m)^{\mathsf{H}}} \tag{19}$$

$$\hat{\mathbf{p}}_n = \frac{1}{K} \mathbf{U}^{(m)} \mathbf{s}^*, \tag{20}$$

where $\mathbf{s} = [s_{\text{SOI}}(k), s_{\text{SOI}}(k-1), \ldots, s_{\text{SOI}}(k - K + 1)]^{\mathsf{T}} \in \mathbb{C}^{K \times 1}$. The mth order subfilter updating rule is given by $\hat{\mathbf{w}}_m = \hat{\mathbf{R}}_m^{-1} \hat{\mathbf{p}}_m$. The subfilters are estimated in an alternate fashion until convergence, which is attained when the error between two consecutive iterations is smaller than a threshold ε. This procedure is described in Algorithm 1.

The multilinear MMSE beamforming algorithm presents a computational complexity of $\mathcal{O}\left(Q \sum_{m=1}^M N_m^2\right)$, where Q is the number of iterations necessary to attain the convergence. Such an alternating minimization procedure has a monotonic convergence. In this work, we do not assume any prior knowledge on the array response and a random initialization is used. In the chosen array configurations, convergence is usually achieved within 4 or 6 iterations. It is worth mentioning that an analytical convergence analysis of this algorithm is a challenging research topic which is under investigation.

An alternative approach to solve (16) would consist in using a gradient-based algorithm. The idea of this algorithm is similar to that of [16], therein referred to as TensorLMS. However, such an approach would need small step sizes and convergence can be much slower in comparison with the multilinear MMSE algorithm.

Algorithm 1 Multilinear MMSE

procedure MULTILINEARMMSE($\mathcal{X}$, $\mathbf{s}$, ε)
  $q \leftarrow 1$
  Initialize $e(q)$, $\mathbf{w}_m(q)$, $m = 1, \ldots, M$.
  repeat
    for $m = 1, \ldots, M$ **do**
      Calculate $\mathbf{U}^{(m)}(q)$ using Equation (18)
      $\hat{\mathbf{R}}_m \leftarrow (1/K)\mathbf{U}^{(m)}\mathbf{U}^{(m)^{\mathsf{H}}}$
      $\hat{\mathbf{p}}_m \leftarrow (1/K)\mathbf{U}^{(m)}\mathbf{s}^*$
      $\mathbf{w}_m(q+1) \leftarrow \hat{\mathbf{R}}_m^{-1}\hat{\mathbf{p}}_m$
    end for
    $q \leftarrow q + 1$
    $\mathbf{y}(q) \leftarrow \mathcal{X} \times_1 \mathbf{w}_1(q)^{\mathsf{H}} \ldots \times_M \mathbf{w}_M(q)^{\mathsf{H}}$
    $e(q) = \|\mathbf{s} - \mathbf{y}(q)\|_2^2/K$
  until $|e(q) - e(q-1)| < \varepsilon$
end procedure

4. NUMERICAL RESULTS

Computer experiments were conducted in order to assess the SOI estimation performance and the computational complexity of the proposed tensor beamformer. In this context, $R = 3$ uncorrelated QPSK signals with unitary variance arriving from the directions (θ_r, ϕ_r) rad $\in \left\{\left(\frac{\pi}{3}, -\frac{\pi}{4}\right), \left(\frac{\pi}{6}, \frac{\pi}{3}\right), \left(\frac{\pi}{4}, -\frac{\pi}{6}\right)\right\}$ were considered. The signal corresponding to $r = 1$ was set as SOI. The linear MMSE beamformer (9) was used as benchmark method. Recall that the linear beamformer ignores the multilinear translation invariant structure of the sensor array, by operating over the vectorized form of the received signal tensor. A noise component was added to the observed signals at the array and the signal-to-noise ratio was set to $15\,\mathrm{dB}$. The convergence threshold of the multilinear MMSE algorithm was set to $\varepsilon = 10^{-6}$. The mean performance indices were calculated by averaging the results obtained in $J = 100$ Monte Carlo (MC) realizations. The SOI estimation performance was evaluated in terms of the MSE measure, defined as $\mathrm{MSE} = \frac{1}{J}\sum_{j=1}^{J}\frac{1}{K}\|\mathbf{s}^{(j)} - \langle\mathcal{X}^{(j)}, \mathcal{W}^{(j)^*}\rangle\|_2^2$, where the superscript $(\cdot)^{(j)}$ denotes the jth MC realization. The number of FLOPS demanded by each method was computed using the Lightspeed MATLAB toolbox [17]. The Tensorlab toolbox [18] was used to implement the tensor operations involved in the proposed algorithm.

Two simulation scenarios were considered. In the first one, the performance indices were calculated by varying the number N of sensors of the global array for $K = 5000$ samples, as depicted in Fig. 2. In the second scenario, the performance indices were calculated by varying the sample size K, as illustrated in Fig. 3. In this case, the global array consisted of $N = 128$ sensors. In both scenarios, the global array was formed by translating a 2×4 uniform rectangular array, the reference array, along the x-axis.

The left plots of Figures 2 and 3 show that the multilinear MMSE algorithm offers a reduced computational complexity compared to the linear (vector) MMSE filter thanks to the exploitation of the array separability, as expected. The gains are particularly more pronounced for $M = 3$ and 4. On the other hand, the right plots of these figures indicate that the MSE of the proposed algorithm is $0.5\,\mathrm{dB}$ above that of the linear beamformer. Such a performance gap can be considered negligible in view of the computational gains, and it can be explained due to the loss of optimality of the rank-1 filter. Therefore, the multilinear algorithm offers a reduction on the computation cost with almost no trade-offs in terms of MSE performance for multilinear translation invariant sensor arrays.

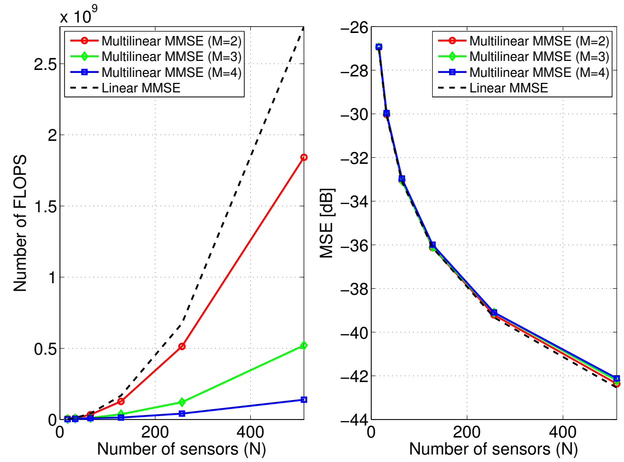

Fig. 2. Performance for a varying number of sensors for $K = 5000$ samples.

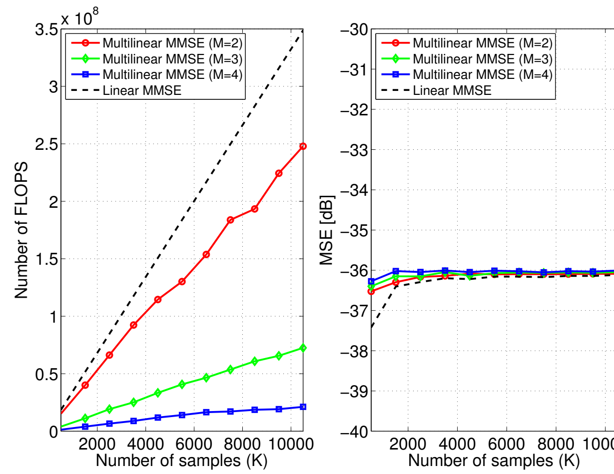

Fig. 3. Performance for a varying sample size for $N = 128$ sensors.

5. CONCLUSION AND PERSPECTIVES

There has been a growing interest on array processing systems capable of processing data received by a massive number of sensors. Multilinear array models are interesting in this context since they represent a sensor array in simpler terms, allowing the development of computationally efficient array processing methods. In this work, a tensor beamformer that exploits the separability present in multilinear translation invariant arrays model was presented. Numerical results showed that the presented method has a reduced processing time with almost no performance loss compared with the linear beamforming solution that operates on the global array by ignoring the array manifold separability. A future work includes the extension of the proposed tensor beamforming to the wideband filtering scenario, where separability can be further exploited in the joint space-time domain. In this work, we have adopted a rank-1 representation for the beamforming filter. The use of low-rank tensor beamformers will be addressed in the future.

6. REFERENCES

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