Enhanced Block Term Decomposition for Atrial Activity Extraction in Atrial Fibrillation ECG

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Abstract—**Atrial fibrillation (AF) is the most prevalent sustained cardiac arrhythmia but is still considered a challenging research subject since its electrophysiological mechanisms are not yet fully understood. Analyzing the atrial activity (AA) signal observed in surface electrocardiograms (ECG) is useful for clinical management and better understanding the propagation mechanisms inside the atria, but ventricular activity (VA) masks the AA in time and frequency domains. Signal processing techniques have been used to extract the AA signal. Blind Source Separation (BSS) methods can accomplish this task from multilead ECG. Recently, a deterministic tensor-based BSS method based on the Block Term Decomposition (BTD) was proposed and offered promising results in AA estimation. This method assumes that AF ECG leads can be expressed as linear combinations of damped exponential sources. However, QRST complexes of VA do not match this model, causing numerical issues. The present contribution proposes a Principal Component Analysis (PCA) preprocessing stage to attenuate the ventricular components. Experimental results show that this stage alleviates the ECG model mismatch, resulting in better AA estimation compared to competing methods and improved numerical properties compared to BTD without preprocessing.**

I. Introduction

Signal processing is a fundamental tool in the study of cardiac electrophysiology. Electrocardiogram (ECG) signal processing methods aim at extracting features that provide insights into the heart's conditions. Besides being useful for clinical management, these features aid researchers to better understand the electrophysiological mechanisms of heart diseases. Atrial fibrillation (AF) is the most prevalent sustained cardiac arrhythmia, and has been attracting increasing research attention, because its genesis and propagation mechanisms are not yet completely understood. AF consists of disorganized electrical activation of the atria caused by ectopic sources around the pulmonary veins and the propagation of multiple self-sustaining wavelets. In AF ECG, these wavelets are reflected as the fibrillatory waves (f-waves) that replace the P-wave of normal atrial activation. Spectral features of f-waves, such as the dominant frequency (DF), are thought to correlate with the atrial tissue refractoriness, thus providing knowledge on AF physiological properties. However, AA is masked by the QRST complex of ventricular activity (VA) at each heartbeat, as illustrated in Figs. 1(a) and 4(a). Since masking also occurs in the frequency domain, signal processing techniques are necessary to properly estimate the AA before further analysis.

Average Beat Subtraction (ABS) and Blind Source Separation (BSS) methods are among the most popular approaches to noninvasive AA extraction [1]–[3]. In [4], [5], we have introduced a deterministic tensor-based BSS method based on the Block Term Decomposition (BTD) [6]. As opposed to ABS and classical BSS techniques, this tensor approach offers the possibility of processing short data records. The method assumes that AF ECG leads can be expressed as linear combinations of damped exponential sources, which is a plausible assumption due to the quasi-harmonic structure of the fibrillatory waves. Source separation is then performed by computing the BTD of the ECG data tensor, obtained by mapping the ECG leads onto Hankel matrices and arranging them in a third-order tensor. Computer experiments showed that AA could be extracted provided that the decomposition parameters were correctly selected. The experiments also showed that block terms with large multilinear rank were necessary to separate the AA from VA, resulting in numerical issues such as slow convergence and large decomposition residual error. This phenomenon could mainly be explained by the sharp peaks of VA, as shown in Figs. 1(a) and 4(a), which do not match the assumed model, producing high-rank block terms.

The present contribution employs signal subspace methods to attenuate the VA on multilead AF ECG as a preprocessing stage prior to BTD computation. This preprocessing mitigates the effects of model mismatch on the BTD step. We investigate the performance of Principal Component Analysis (PCA) in this preprocessing task. Next, the applicability of this improved BTD is evaluated on real AF ECG by comparing its performance to those of two benchmark methods: Adaptive Singular Value Cancellation (ASVC) [7] and RobustICA-f [8].

II. Methods

A. Database and Signal Acquisition

Standard 12-lead ECG signals were recorded on two patients diagnosed with persistent AF at the Cardiology Department of Princess Grace Hospital, Monaco. These recordings were acquired at a sampling rate of 977 Hz and lasted about 60 s each. ECG were processed by a forward-backward type-II Chebyshev bandpass filter with cut-off frequencies of 0.5 Hz and 30 Hz to remove baseline wander and powerline interference. The resulting signals were stored in matrix $\mathbf{Y} \in \mathbb{R}^{12\times N}$, where N denotes the sample size. Simultaneous invasive electrogram (EGM) recordings were acquired by placing bipolar catheters inside the left atrial appendage (LAA).

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B. Ventricular Activity Attenuation

In this contribution, we propose to process the ECG recordings using PCA to mitigate the VA interference before tensor analysis. PCA is a multivariate technique that decomposes data into uncorrelated variables called principal components [9], which are sorted according to their contribution to data variance. Since VA clearly contributes to most of the variance of AF ECG, the first principal components are usually related to the QRST waves. To attenuate these peaks in AF ECG, the ECG data are centered, and then the observation matrix is decomposed as

$$\mathbf{Y} = [\mathbf{h}_1, \dots, \mathbf{h}_{12}]\,[\mathbf{p}_1, \dots, \mathbf{p}_{12}]^\mathsf{T}$$

where $\mathbf{p}_k \in \mathbb{R}^{N\times 1}$ denotes the kth principal component and $\mathbf{h}_k \in \mathbb{R}^{12\times 1}$ its spatial signature for $k = 1, \dots, 12$. The principal components are visually inspected to identify the K ventricular components to be discarded together with their corresponding spatial signature, and then the ECG is rebuilt as $\tilde{\mathbf{Y}}$. For instance, if the first three principal components were discarded ($K = 3$), then:

$$\tilde{\mathbf{Y}} = [\mathbf{h}_4, \dots, \mathbf{h}_{12}]\,[\mathbf{p}_4, \dots, \mathbf{p}_{12}]^\mathsf{T}.$$

PCA is tantamount to computing the SVD of a data matrix $\mathbf{Y}$.

C. Block Term Decomposition

In general, due to orthogonality constraints on its factors, PCA is unable to perform a successful AA extraction, but can at least attenuate the VA interference. Once the AF ECG signals have been preprocessed as described above, rank-$(L, L, 1)$ BTD [10] is employed to extract the AA from them, as originally proposed in [4], [5]. Symbol L denotes the rank of the mode-1 and mode-2 matricizations of each tensor term in the decomposition. In order to reduce the computational cost of BTD, ECG is downsampled by a factor $D \in \mathbb{N}$, yielding the ECG matrix $\tilde{\mathbf{Y}}_d \in \mathbb{R}^{12\times N'}$, where $N' = N/D$ is the new sample size. We assume the linear instantaneous mixture model for ECG signals [2]: $\tilde{\mathbf{Y}}_d = \mathbf{MS}$, where $\mathbf{M} \in \mathbb{R}^{12\times R}$ denotes the mixing matrix, $\mathbf{S} \in \mathbb{R}^{R\times N'}$ the source matrix with R sources, and the measurement noise is ignored for simplicity. ECG is tensorized by mapping each row of $\tilde{\mathbf{Y}}_d$ onto an $(I \times J)$ Hankel matrix and arranging the resulting matrices along the third mode of tensor $\mathcal{T} \in \mathbb{R}^{I\times J\times 12}$, where $I + J - 1 = N'$, with $I = J = (N' + 1)/2$ if N' is odd, or $I = N'/2$ and $J = N'/2 + 1$, otherwise. It can be shown that the rank-$(L, L, 1)$ BTD of $\mathcal{T}$ can be expressed as

$$\mathcal{T} = \sum_{r=1}^{R} \mathbf{H}_\mathbf{S}^{(r)} \circ \mathbf{m}_r \qquad (1)$$

where "$\circ$" denotes the outer product, $\mathbf{m}_r \in \mathbb{R}^{12\times 1}$ the rth column of $\mathbf{M}$, and $\mathbf{H}_\mathbf{S}^{(r)} \in \mathbb{R}^{I\times J}$ the rank-L truncated Hankel matrix associated with the rth source [4], [5]. The elements of $\mathbf{H}_\mathbf{S}^{(r)}$ are given by $[\mathbf{H}_\mathbf{S}^{(r)}]_{i,j} = s_{r,i+j-1}$ for $i = 1, \dots, I$ and $j = 1, \dots, J$. The decomposition (1) was shown to be essentially unique whenever the R sources are modeled as sums of L damped exponential signals [10]. In this case, the elements of $\mathbf{S}$ are given by:

$$[\mathbf{S}]_{r,n} = \sum_{\ell=1}^{L} c_\ell z_{\ell,r}^{n-1}, \quad 1 \le r \le R,\, 1 \le n \le N' \qquad (2)$$

where c_ℓ is a scalar factor, and $z_{\ell,r}$ is the ℓth pole of the rth source. Indeed, this is a plausible signal model for atrial activity during AF, due to its typical quasi-harmonic structure [1].

Before calculating (1), the model parameters R and L must be selected. To the best of our knowledge, currently there are no automatic model selection methods for BTD. Therefore, the model parameters are selected following Occam's razor principle: the simplest (R, L) combination that provides proper AA extraction is selected, as will be detailed in Sec. III. The tensor model (1) is formulated as a Structured Data Fusion (SDF) problem [11] and is solved using Tensorlab's SDF nonlinear least-squares (SDF-NLS) implementation [12]. The BTD factors are initialized as matrices containing values drawn from a zero-mean, unit-variance Gaussian distribution. The convergence threshold parameter is set to 10^{-10}. Once (1) is computed, the R sources' waveforms are estimated by averaging the antidiagonals of $\mathbf{H}_\mathbf{S}^{(r)}$ for $r = 1, \dots, R$ [10].

D. Fourier Analysis

Afterwards, the estimated BTD sources are interpolated back to 977 Hz, and then the AA is identified by transforming the separated sources into the frequency domain. We choose the source with DF between 3 Hz and 9 Hz maximizing the spectral concentration (SC), defined as in [3]:

$$SC = \frac{\int_{0.82DF}^{1.17DF} P_r(f)df}{\int_{0}^{F_s/2} P_r(f)df}$$

where P_r denotes rth source Welch's power spectral density estimate (4096-point FFT, 2048-bin Hamming window, 50% overlap), and F_s and the sampling frequency. If no source lies between this interval, then the source with the greatest SC is chosen. To evaluate the correlation of dominant frequency between the estimated AA and the intracardiac recordings, the EGM is processed using Botteron's rectification method [13].

III. Experimental Results

Some experiments are conducted to assess the influence of PCA on the convergence properties of the SDF-NLS algorithm used to compute the rank-$(L, L, 1)$ BTD. Two experimental scenarios are considered: a patient with strong AA amplitude and another patient with weak AA amplitude. In each scenario, the remaining cumulative percentage of variance σ^2, the estimated DF, the number of iterations to convergence, and the residual error ρ are computed for BTD using PCA preprocessing (PBTD) as functions of K, the number of discarded dominant components. The residual error is defined as $\rho = \frac{1}{2}\|\mathcal{T} - \hat{\mathcal{T}}\|_\mathrm{F}^2$, where $\|\cdot\|_\mathrm{F}$ denotes the Frobenius norm, and $\hat{\mathcal{T}}$ the BTD of tensor $\mathcal{T}$ after convergence [11]. Parameter K is then selected by minimizing the difference between the estimated DF and that of EGM recordings. PBTD's performance is compared to that of RobustICA-f, ASVC, and BTD without preprocessing.

Each experiment consists of 100 independent realizations where PBTD is randomly initialized, as described in Section II-C. The results depicted in Figs. 1-6 correspond to the final realization and are similar to those of the other realizations. In both scenarios, ECG segments of 5 s are selected from the original recordings and downsampled by a factor of $D = 10$ to reduce BTD's computational cost. The number of maximum allowed SDF-NLS iterations is set to 1000.

For the first scenario, lead V1 is shown in Fig. 1(a). From the 12-lead ECG, the first five principal components are depicted in Fig. 2. The experiment results shown at the top of Table I are obtained for $(R, L) = (2, 8)$. According to these results, the mean DF of the AA extracted for $K = 2$ is the closest to 8.82 Hz, the DF of the LAA EGM. Hence, $K = 2$ was chosen to generate the results depicted in Figures 1 and 3.

Figure 5 shows the first five ECG principal components extracted in the second scenario. The results shown at the bottom of Table I are obtained for $(R, L) = (3, 8)$. For $K = 4$, PBTD provides estimates with DF close to 5.57 Hz, the DF of the LAA EGM. The results shown in Figures 4 and 6 are obtained for $(R, L, K) = (3, 8, 4)$. BTD without preprocessing ($K = 0$, not shown in Table I) failed to extract the AA in both scenarios for many (R, L) combinations. The extracted sources contained mostly ventricular components. In this case, a shorter observation window and larger values for R and L would be required as in [5] to successfully extract the AA.

IV. Discussion

The conducted MC experiments show that suitably discarding principal components can have a beneficial effect on BTD's AA extraction performance. In the first scenario, AA is mostly concentrated in the third and fourth principal components, as depicted in Figure 2. This is due to the high AA amplitude in the ECG. The results in Table I suggest that BTD's DF estimation accuracy deteriorates when VA is not sufficiently attenuated (low values of K) or when the components containing most of AA are rejected ($K = 3, 4$). In spite of that, the numerical performance of SDF-NLS generally increases with K, since fewer spurious components lead to decreased model mismatch. In the second scenario, however, AA is mostly concentrated on the fifth principal component, whereas VA is concentrated on the first four principal components, as it can be seen in Figure 5. This is because AA has low amplitude compared to VA. The results in Table I indicate that AA is properly estimated when those VA-related components were rejected ($K = 4$).

Once preprocessing is performed, model parameters that yield low multilinear rank block terms can be chosen. In the first scenario, $(R, L) = (2, 8)$ is the simplest combination that provides proper AA extraction. However, other combinations leading to more complex models also provide satisfactory performance. Lower L values yield AA estimates with larger SC but fail to capture the second harmonic around 12 Hz depicted in Figure 3. By contrast, larger L values allow BTD to retrieve more signal components, which ameliorates the estimation fidelity, but also tends to capture ventricular artifacts. In the second scenario, the number of poles L produces the same effect on the estimated AA as in the first scenario. However, the second harmonic could be retrieved only for $R = 3$. Therefore, the number of poles controls the harmonic structure of the estimated AA signal, as it could be expected from Equation (2).

PBTD provides AA estimates more robust to ventricular interference than RobustICA-f and ASVC given that (R, L, K) is properly selected. Although the estimated waveforms by PBTD in Figures 1 and 4 are not the most similar to the AA in the TQ segments, their spectra reveal their atrial origin. We observe that the AA estimates provided by PBTD properly reject the interference in the frequency domain. This is because the damped exponential model rejects non-conforming interference, while preserving the AA quasi-harmonic structure. BTD without preprocessing does not work in the conducted experiments due to large model mismatch, mostly caused by ventricular artifacts. A smaller sample size is necessary to decrease this mismatch as suggested in [4], [5].

TABLE I
Top: First scenario results, where EGM DF is 8.82 Hz.
Bottom: Second scenario results, where EGM DF is 5.57 Hz.

K	σ^2 [%]	DF [Hz]	Iterations	ρ [$\times 10^{-5}$]
1	51.43	3.95 ± 1.84	491 ± 201	422 ± 0.1
2	20.55	6.41 ± 0.26	342 ± 98	51.6 ± 4.6
3	11.66	5.74 ± 0.06	176 ± 109	16.4 ± 0.0
4	6.88	1.34 ± 2.74	108 ± 50	6.9 ± 0.8
5	3.93	3.15 ± 3.89	45 ± 19	2.2 ± 0.7
6	2.27	3.23 ± 3.89	40 ± 12	0.7 ± 0.2
1	47.28	2.59 ± 0.17	1000	279 ± 1.5
2	22.84	5.01 ± 6.72	476 ± 137	66.5 ± 0.1
3	8.88	6.50 ± 2.22	188 ± 97	6.8 ± 1.7
4	4.37	5.96 ± 0.05	76 ± 26	1.5 ± 0.3
5	2.18	3.29 ± 2.92	35 ± 8	0.5 ± 0.1
6	1.04	2.16 ± 2.06	13 ± 8	0.1 ± 0.0

V. Conclusion

We have proposed a subspace-based preprocessing stage to attenuate high-rank ventricular components in AF ECG that do not conform to the damped exponential model assumed in BTD based on Hankel matrices. Attenuating the ventricular components while preserving the atrial components can have a beneficial effect on BTD's performance. The filtered ECG recordings are more adapted to the assumed model, allowing BTD to extract the AA more efficiently than the competing methods provided that the number of block terms and their multilinear rank have been properly selected. In the examples presented in this work, the proposed method proves more robust to ventricular interference that hinders AA estimation, and provides estimates less contaminated by artifacts than RobustICA-f and ASVC. Research perspectives include a thorough comparative analysis of these techniques in a full AF ECG database.

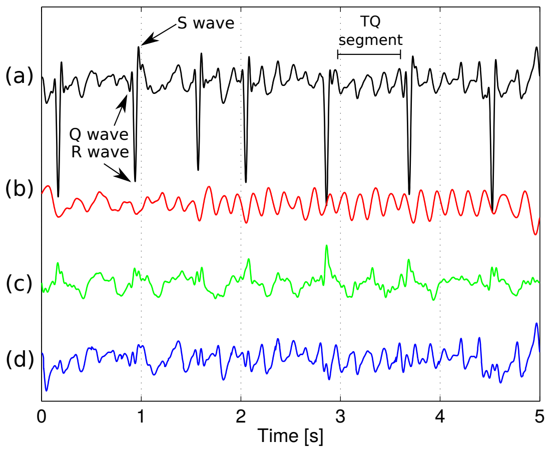

Fig. 1. Results on the first scenario. (a) Original lead V1. (b) Estimated AA from PBTD for $(R, L, K) = (2, 8, 2)$, (c) RobustICA-f, and (d) ASVC.

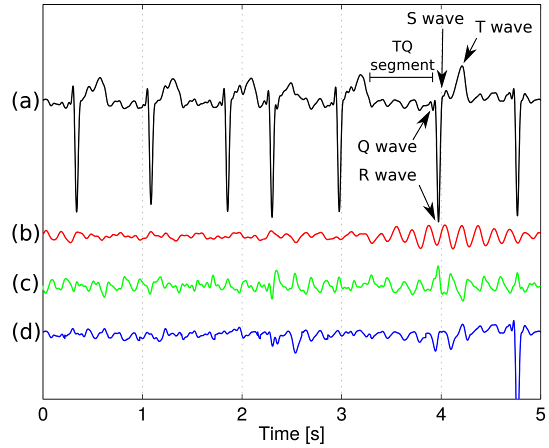

Fig. 4. Results on the second scenario.(a) Original lead V1. (b) Estimated AA from PBTD for $(R, L, K) = (3, 8, 4)$, (c) RobustICA-f, and (d) ASVC.

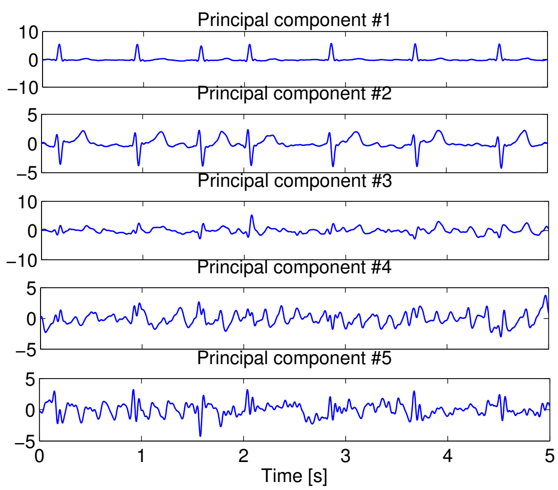

Fig. 2. First five ECG principal components in the first scenario.

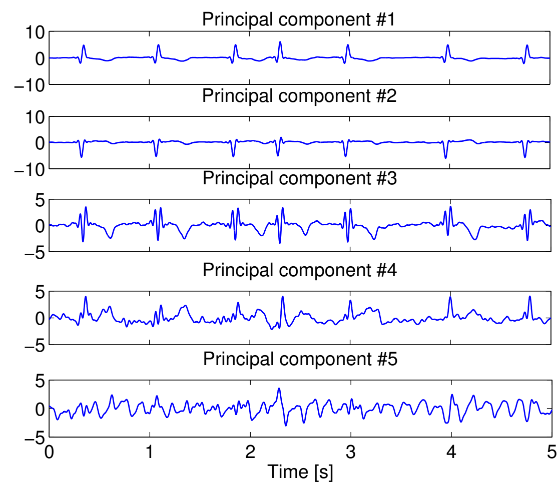

Fig. 5. First five ECG principal components in the second scenario.

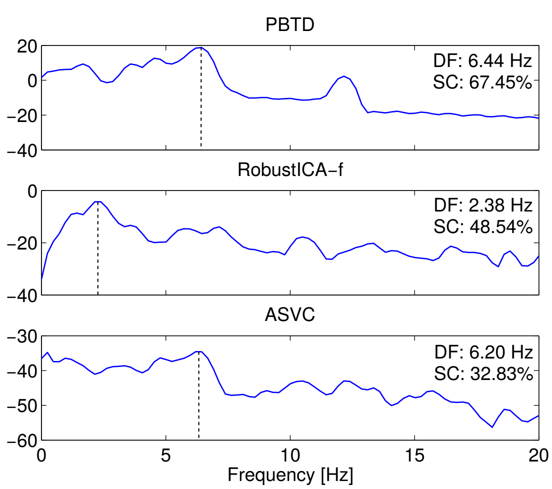

Fig. 3. Power spectral density of the (Top) PBTD, (Middle) RobustICA-f, and (Bottom) ASVC AA estimates in the first scenario (EGM DF $= 8.82$ Hz).

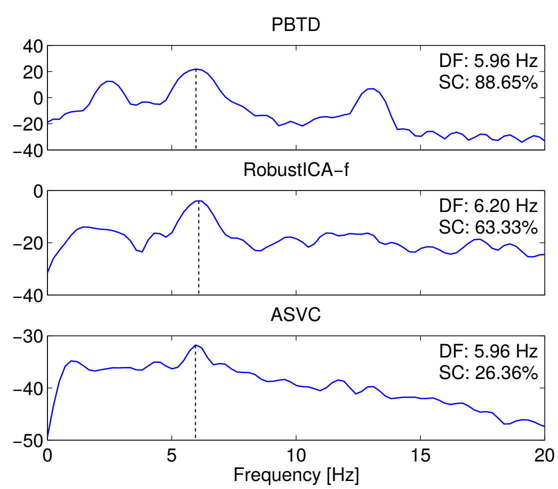

Fig. 6. Power spectral density of the (Top) PBTD, (Middle) RobustICA-f, and (Bottom) ASVC AA estimates in the second scenario (EGM DF $= 5.57$ Hz).

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