

Comparison of Floquet Port-Based Unit Cell Design and Characteristic Mode Analysis for Anomalous Reflecting Metasurfaces

Axel Hoffmann*, Michael Ponschab[†], Mathias Pietzka[†],
Lucas N. Ribeiro[†], Philipp K. Gentner[†], Dirk Manteuffel*

*Institute of Microwave and Wireless Systems, Leibniz University Hannover, Hannover, Germany

[†]Ericsson Antenna Technology GmbH, Rosenheim, Germany

E-mails: hoffmann@imw.uni-hannover.de, michael.ponschab@ericsson.com

Abstract—A comparison of Floquet port-based design and Characteristic Mode Analysis (CMA) for anomalous reflecting metasurfaces is presented. Floquet port simulations of a unit cell are performed to realize the desired induced phase gradient over the surface. While this method is well established and implemented in commercial software, it offers little insight into the electromagnetics of the structure. Therefore, a CMA utilizing the Floquet theorem is linked to the structure analysis to give further insight into the current distribution of the unit cell. An example setup as well as computational results are provided to compare both methods. It is shown that the CMA method provides advanced analysis of the physical behavior of the reflecting metasurface, which helps to improve the design process.

Index Terms—periodic surfaces, reflecting intelligent surfaces, characteristic modes, computational electromagnetics

I. INTRODUCTION

The sixth generation (6G) of mobile communication systems is expected to improve the performance indicators provided by 5G systems and define various new use cases [1]. The deployment of controllable reflectors has been put forward as one of the main enablers of 6G use cases [2]. These reflective structures can influence the propagation environment such that the system performance is improved. In particular, controllable reflectors extend coverage in non-line of sight (NLOS) propagation conditions [3], increase the multiple-input multiple-output (MIMO) channel rank distribution in NLOS [4], and enhance the performance of interference cancellation of multi-user MIMO schemes [5]. Compared to classical relaying stations, these reflectors are expected to be inexpensive and cost efficient.

Controllable reflectors should have an anomalous behavior to influence the propagation environment, i.e., the incident and reflection angles should be different. This can be achieved by metasurface-based reflectors. They consist of high-frequency structures on printed circuit boards that enable amplitude and phase tapers of an incident locally plane wavefront. Such structures can be static or dynamically reconfigurable according to their complexity and the network requirements.

Metasurfaces can be designed with various approaches. For instance, a characteristic mode analysis (CMA)-based design

has been presented in [6], [7]. Traditionally, periodic structures have also been designed considering Floquet theory [8]. Therefore, a natural question that arises is: how do these methods compare to each other? What are the strong and weak points of these methods? This contribution provides an insight into this question by considering the design of an anomalous reflector.

This paper is organized as follows. The theoretical framework is presented in Section II. Based thereon, the Floquet-based simulation is depicted in Section III. This is then followed by the description of the application of the CMA in Section IV. Finally, the comparison between both methods is presented and discussed in Section V.

II. THEORETICAL FRAMEWORK

To implement a phase taper over a reflecting surface, the so-called local reflection coefficient at position vector $\mathbf{r}$

$$R(\mathbf{r}) = \frac{Z_S(\mathbf{r}) - Z_0}{Z_S(\mathbf{r}) + Z_0} = e^{j\Phi(\mathbf{r})} \quad (1)$$

linking the reflection phase $\Phi(\mathbf{r})$ to the surface impedance Z_S , can be applied [9], wherein Z_0 is the wave impedance of the incident wave and j is the imaginary unit. In order to take the expected direction of the incident wave θ_i and the desired direction for the reflected wave θ_r into account, the surface impedance can be written as [9]:

$$Z_S(\mathbf{r}) = \frac{Z_0}{\sqrt{\cos \theta_i} \cos \theta_r} \frac{\sqrt{\cos \theta_r} + \sqrt{\cos \theta_i} e^{j\Phi(\mathbf{r})}}{\sqrt{\cos \theta_i} - \sqrt{\cos \theta_r} e^{j\Phi(\mathbf{r})}}. \quad (2)$$

Considering the required 2π -periodic reflection phase $\Phi(\mathbf{r})$, it follows for the impedance $Z_S(\mathbf{r}) = Z_S(\mathbf{r} + \mathbf{D})$, with the period $\mathbf{D} = (D_x, D_y)^T$ depending on the wave length λ [9]. For a linear y -polarization, the period is

$$D_x = \frac{\lambda}{\sin \theta_r - \sin \theta_i}. \quad (3)$$

In turn, for an x -polarized incident wave the period D_y behaves vice versa.

When designing a surface corresponding to the periodic surface impedance Z_S , the use of boundary conditions utilizing Floquet theory simplifies the design process. This is due to the

possibility to evaluate the whole structure shown in Fig. 1, with a single unit cell of dimensions $D_x \times D_y$. In this case, the surface current density $\mathbf{J}(\mathbf{r})$ at each point $\mathbf{r}$ can be expressed in terms of the phase shift induced by the periodicity $\mathbf{D}$. This includes the influence of the entire structure into the solution of a single unit cell, such that the current density can be expressed as

$$\mathbf{J}(x + D_x, y + D_y) = e^{-jk_i(D_x + D_y)} \mathbf{j}(x, y). \quad (4)$$

The Floquet modes are 2D spectral series defined as [10]

$$\mathbf{j}(\mathbf{r}) = \sum_{m,n} \tilde{j}_{m,n} e^{jk_{mn}\mathbf{r}} \quad (5)$$

$$\tilde{j}_{m,n} = \frac{1}{D_x D_y} \int \mathbf{j}(\mathbf{r}) e^{-jk_i\mathbf{r}} d\mathbf{r} \quad (6)$$

with the wave number $\mathbf{k}$.

It is usual in metasurface design to consider Floquet boundary conditions, most commonly in connection with frequency-domain solvers. In Dassault Systems CST Microwave Studio, this evaluation is carried out based on S parameters utilizing ‘‘Floquet ports’’.

While the Floquet port simplifies the design process by limiting the simulation to a single unit cell, it does not offer much insight into the electromagnetics of the unit cell itself and it behaves in an array. On the other hand, the Theory of Characteristic Modes [11] has been proven to enable an efficient tool for the analysis and design of electromagnetic structure [6], [7]. The generalized eigenvalue equation defining the characteristic modes (CM) with $\mathbf{B} = \mathbf{X}$ and $\mathbf{A} = \mathbf{R}$ is defined as [11]

$$\mathbf{B}\mathbf{I}_\nu = u_\nu \mathbf{A}\mathbf{I}_\nu \quad (7)$$

with $\mathbf{I}_\nu$ being the ν th characteristic surface current density vector and u_ν the corresponding generalized eigenvalue. The total surface current density may then be expressed as [11]:

$$\mathbf{I} = \sum_{\nu} a_{\nu} \mathbf{I}_{\nu} = \sum_{\nu=1}^N \frac{\mathbf{I}_{\nu}^H \mathbf{V}^i \mathbf{I}_{\nu}}{1 + ju_{\nu}}. \quad (8)$$

Here, a_{ν} denotes the normalized modal weighting coefficient, $\mathbf{V}^i$ the excitation vector, and $(\cdot)^H$ the conjugate transpose operator. By setting the excitation vector to

$$\mathbf{V}^i = \psi^H \mathbf{E}^i \quad (9)$$

containing the normalized impressed electric field $\mathbf{E}^i$ and the base function ψ_n , a method of moments (MoM) system of equations $\mathbf{V}^i = -\mathbf{Z}\mathbf{I}$ can be constructed [11].

The CMA of a metasurface is based on the complex-valued surface impedance matrix $\mathbf{Z}_S = \tilde{\mathbf{R}} + j\tilde{\mathbf{X}}$ [10], [12]. Considering an infinite array of unit cells, the excitation vector

$$\mathbf{V}^i = -\frac{1}{D_x D_y} \mathbf{I} \sum_{\gamma} \mathbf{f}_{\gamma}^H \mathbf{Z}_{\gamma} \mathbf{f}_{\gamma} \quad (10)$$

depends on the spectral basis function matrix $\mathbf{f}_{\gamma}$ and the spectral impedance dyad $\mathbf{Z}_{\gamma}$, with the spectral index γ [10].

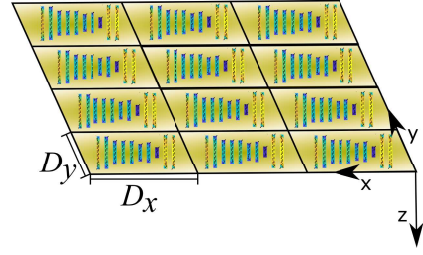


Fig. 1: Periodic surface with current densities in unit cells of dimensions $D_x \times D_y$.

Based thereon, it is necessary to adapt the impedance matrix decomposition criterion. It is no longer defined by the decomposition into imaginary part ($\mathbf{X}$) and real part ($\mathbf{R}$) of $\mathbf{Z}_S$ but on the radiating ($\mathbf{A} = \tilde{\mathbf{R}}$) and evanescent ($\mathbf{B} = \tilde{\mathbf{X}}$) contribution of the Floquet harmonics. The evaluation therefore takes place on the dyad $\mathbf{Z}_{m,n}$ [10]

$$\mathbf{Z}_{S,m,n} = \begin{cases} \tilde{\mathbf{R}}_{m,n}, & \text{for } \text{Im}(\mathbf{Z}_{\gamma}) = 0 \\ \tilde{\mathbf{X}}_{m,n}, & \text{for } \text{Re}(\mathbf{Z}_{\gamma}) = 0 \end{cases}, \quad (11)$$

resulting in complex $\tilde{\mathbf{R}}_{m,n}$ and $\tilde{\mathbf{X}}_{m,n}$. Since $\tilde{\mathbf{R}}$ is Hermitian and $\tilde{\mathbf{X}}$ is anti-Hermitian, u_{ν} remains real valued. In any case, the weighted sum of $\mathbf{I}_{\nu}$ reconstructs the total surface current density [10]. To evaluate the spectral reflection mapping from the incident spectral field γ into the scattered spectral field γ' , the modal reflection coefficient tensor $\Gamma^{\gamma\gamma'}$ can be considered. For some impedance dyad $\mathbf{Z}_{\gamma}$, basis function matrix $\mathbf{f}_{\gamma}$, and surface current vectors $\mathbf{I}_{\nu}$, the tensor is calculated as the following outer product [10]:

$$\Gamma^{\gamma\gamma'} = -\sum_{\nu} \frac{\mathbf{Z}_{\gamma} \mathbf{f}_{\gamma} \mathbf{I}_{\nu}}{D_x D_y} \frac{\mathbf{I}_{\nu}^H \mathbf{f}_{\gamma'}^H}{\mathbf{I}_{\nu}^H \tilde{\mathbf{R}} \mathbf{I}_{\nu} (1 + ju_{\nu})}. \quad (12)$$

III. APPLICATION OF FLOQUET PORT SIMULATION FOR METASURFACE DESIGN

For the Floquet port based design Dassaults Systems CST Microwave Studio is utilized. A single polarized wave with an electrical field $\mathbf{E} = [0, E_y, 0]^T$ at the frequency $f = 10$ GHz is incident on the surface under the angle $\theta_i = 0^\circ$. For our example evaluation, the aim is to design a surface, which reflects this wave towards $\theta_r = 40^\circ$. The surface consists of metal patches on a thin ground backed substrate, as shown in Fig. 2. One unit cell contains 10 patches with constant gap size g in between. The surface impedance gradient over the unit cell is accomplished by a variation of the elements.

In a first step, $\mathbf{Z}_S(\mathbf{r})$ needs to be simulated in dependence of the patch length l . Then, the unit cell width $D_x = 46.6$ mm is calculated from (3). Furthermore, setting $g = 1$ mm results in a patch width $w = 3.66$ mm.

A simulation model of a single patch element is solved using Floquet boundary conditions and a Floquet port with phase angle θ_i . Then, a parameter sweep over l is performed. The patches surface impedance is calculated from the reflection parameter and is shown in Fig. 3. The desired phase gradient

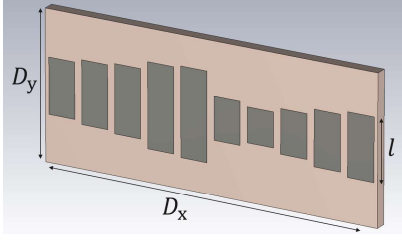


Fig. 2: Unit cell model with patch lengths according to surface impedance gradient.

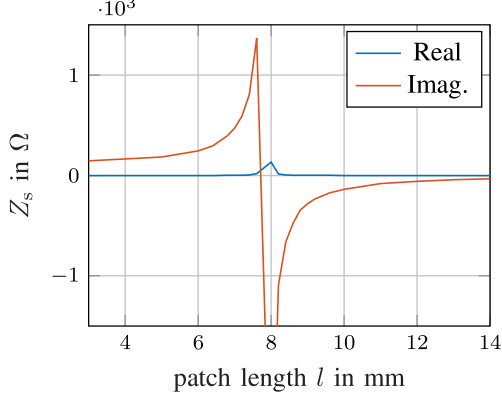


Fig. 3: Surface impedance of a patch element in dependence of its length.

and the surface impedance are calculated analytically from (1) and (2), respectively. The discretization of $Z_S(\mathbf{r})$ at the patch location grid points shown in Fig. 4 allows to adopt the patch lengths in order to fit the desired reactance. A unit cell is formed out of patches of different lengths. Therefore, the coupling between the patches changes slightly compared to the infinite patch array of uniform length. This makes a simulation of the whole unit cell necessary in order to optimize the reflection behavior towards the desired reflection angle θ_r . The unit cell is again simulated with periodic boundaries and

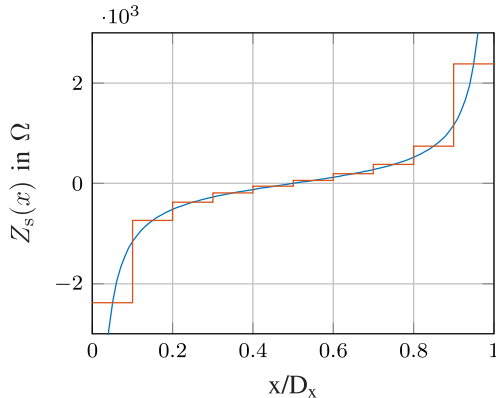


Fig. 4: Desired periodic surface impedance gradient and its discretization on the patch grid points.

a Floquet port. The phase angle for the periodic condition is set to θ_i . As the periodicity of the structure is designed to allow refraction into higher Floquet modes, these must be monitored in addition to the zero order mode. An optimizer is used to adjust the patch lengths and maximize the reflection parameter of Floquet mode $m = 0$ into $m = 1$. The built-in Nelder-Mead-Simplex optimizer is used, which converges in two hundred steps. The resulting unit cell with the length $D_y = 15$ mm is arranged in a 10×10 array generating the reflected field shown in Fig. 8.

IV. APPLICATION OF CHARACTERISTIC MODE ANALYSIS TO METASURFACE DESIGN

For the CMA, an in-house MoM solver developed at the Institute of Microwave and Wireless Systems at Leibniz University of Hannover is used. The design process for reflecting metasurfaces consists of four steps [13]:

- 1) The unit cell dimensions have to be determined according to the polarization, following (3).
- 2) A structure to calculate the MoM on has to be predefined.
- 3) CMA is performed, i.e., the generalized eigenvalue problem based on (7) is solved.
- 4) Finally, the modal currents and fields based on (8) are interpreted with the classic CMA approach to optimize the structure towards the desired reflection behavior.

In order to be consistent with the analysis carried out in section III, a y -polarized wave at the frequency $f = 10$ GHz is used to calculate the unit cell width D_x from (3). For the unit cell length $D_y = \frac{\lambda}{2} = 15$ mm is chosen. Then a structure consisting of 10 conductor backed metal patches is selected to satisfy (2) piecewise (see Fig. 4). Hereafter, the CMs are calculated. At first, a single patch with the width $w = 3.66$ mm is simulated to evaluate the necessary patch length l for the different patches to meet the discretized $Z_S(\mathbf{r})$ by (12). In a next step the whole unit cell (10 patches) is modelled with the beforehand evaluated values for l .

Fig. 5 shows the four most significant modal current densities with the associated modal directivities, weighted by a_ν as shown in Fig. 6a. Analyzing the modal current densities, the two larger inner patches are identified to account mainly for mode 1, whereas mode 2 and mode 3 additionally depend on the two outermost patches on each side. Mode 4 is driven by loop-like currents on all patches. To achieve a higher directivity towards the desired angle θ_r the contributions of mode 2 and mode 3 are to be increased and current densities in directions other than the polarization are to be suppressed. To increase the contribution of mode 2 and mode 3, the two outermost patches are lengthened while keeping the other patches the same length. Additionally the patch width is decreased to omit current densities in x -direction. Then, the CMA of optimized structure is run again. The optimized modal current densities and modal directivities are shown in Fig. 7 and their weighting coefficients a_ν are shown in Fig. 6b. For mode 1 and mode 2, the modal current density now is

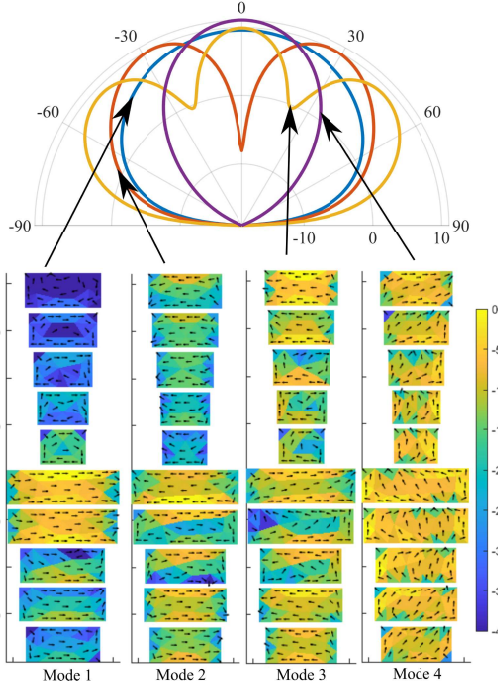


Fig. 5: Θ cut at $\phi = 0^\circ$ of modal directivity in dBi and modal current density, of the initial unit cell set-up.

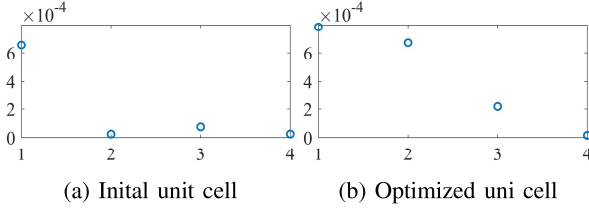


Fig. 6: Modal weighting coefficients of an incident field at $\theta_i = 0^\circ$.

significant only on the opposite outermost patches. Therefore, the modes have different phase center, so that the pattern is slightly tilted. Mode 4 is now no longer driven by loop-like currents but the current density on the two outermost patches on both sides contribute significantly resulting in a pattern with three beams. Arranging the resulting unit cell in a 10×10 array generates the optimized reflected field shown in Fig. 8.

V. METHODOLOGY COMPARISON

The local reflection coefficient [9] is a useful basis for both methods considered in this paper. The Floquet port method can be used to vary the structure towards the desired surface impedance (2). However, it is only based on a comparison of the field contribution of the different Floquet modes based on the reflection coefficient. On the other hand, the Characteristic Mode analysis method requires the additional step of analyzing the weighted far-field pattern and current densities. However, this step enables additional intuitive insight into the electromagnetic design of the unit cell.

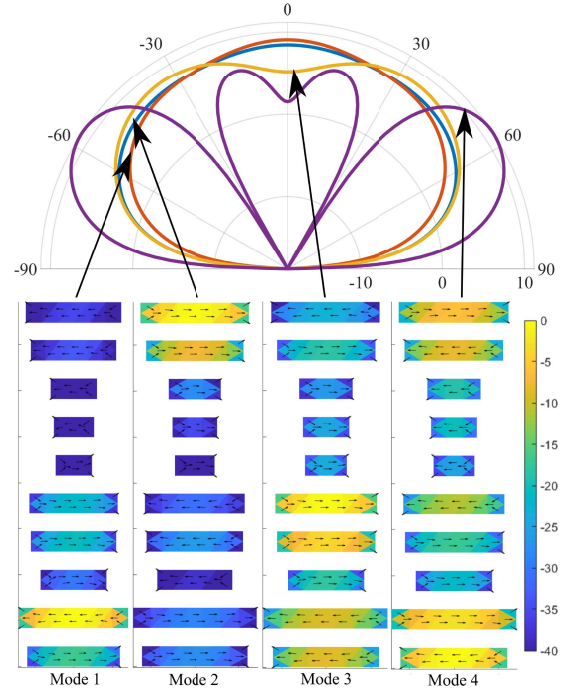


Fig. 7: Θ cut at $\phi = 0^\circ$ of modal directivity in dBi and modal current density, of the unit cell set-up after optimization.

Comparing the resulting far-field pattern of the Floquet port method and the CM method for a 10×10 unit cell array shown in Fig. 8 justifies the additional effort. The radiation of the reflected wave into the desired direction θ_r is increased. While at the same time the reflected wave into specular direction $\theta = 0^\circ$ and retro reflect direction $\theta = -\theta_r$ is decreased respectively. Furthermore, the method yields great insight into the reflection mechanism. It is important to mention the possibility to determine an optimized patch width with the current density directed only in the direction of polarization without breaking the requirements to the reflection. In contrast, the Floquet method provides only an approximation for the patch width by ensuring a sufficiently accurate discretization of (2) to realize a reflected beam into the desired direction. This can be attributed to the fact that, as mentioned in [14], the local reflection coefficient is a near-field approach, while the CM analysis is a far-field design approach and therefore more suited for the design of radiating far-fields.

VI. CONCLUSION

Two different design methods addressing reflecting metasurface are discussed. The state of the art method namely Floquet port simulation is compared to a method applying the CMA. It is shown for an example metasurface that the Floquet port simulation is the most straightforward method but it lacks physical insight. This physical insight can be additionally given by CMA. It is further shown that utilizing the intuitive physical insight, the design can be even improved compared to the Floquet Port based approach. The resulting design offers

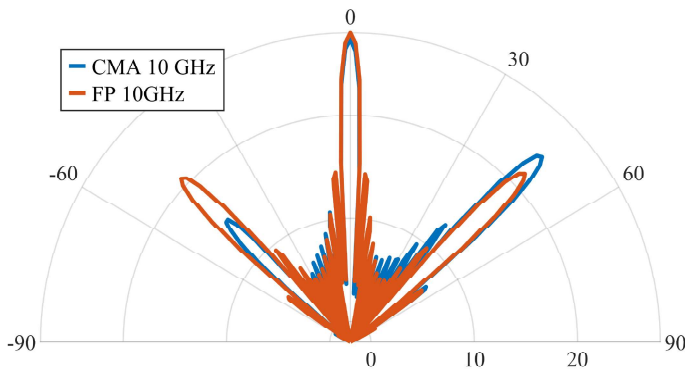


Fig. 8: Θ cut at $\phi = 0^\circ$ of total directivity in dBi of an 10×10 unit cells array. For both methods the reflection into the desired direction θ_r is achieved. With the CM analysis method the retro reflection $-\theta_r$ and specular reflection are better suppressed.

increased directivity towards the desired direction. In future work, the the CMA method can be expanded incorporating dynamic elements to be used in the reflecting intelligent surface design process.

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